THE SCIENCE OF SYSTEMS

A Unified View of Nature's Patterns

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First Edition 2024

ISBN: 979-8-218-35648-4

Acknowledgements

I extend my heartfelt gratitude to my family, friends, and colleagues who have been pillars of support throughout the 13-year journey of crafting this book. Their encouragement has been invaluable, keeping me motivated, and their insightful contributions have played a pivotal role in the refinement of this project. I am profoundly thankful for the meticulous copy-editing assistance provided by Amy Loftus and Elizabeth Kingston. My wife Kyra Shugar, and father Dan Shugar, have provided continual support. I would like the thank Andrew Kornfeld for the design advice and the many conversations that helped shaped the book. Many other people have supported the journey of this book, including Mark Stephens and Howard Kornfeld. Finally, I would like to acknowledge all the thought leaders that have inspired me, and all the thought leaders yet to come for helping humanity understand and solve problems facing our connected and complex world.

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Preface

The central view of systems science—to think in terms of complex connections—offers the necessary perspective to reconcile modern scientific findings and address problems facing the 21st century world. The systems-based view challenges the idea, largely originating in 18th century Western science, that nature is made of disjointed components with collective properties that are simple to predict. Systems theory instead studies nature through a network of relations that can be highly interdependent, chaotic, and complex. This book conveys the essential insights of systems science in three parts.

Part I – Foundations: First, an introduction is provided to system science, which studies all types of systems spanning formal logic as well as natural and social sciences. In its most general form, a system consists of a set of elements and relations. Systems are used to define simple to complex formal models, like logic and math, that can be used to study nature's patterns across emergent levels.

Part II – Theory: The next section provides a history of natural sciences and introduces common patterns in nature, including equilibrium, flux, symmetry, fractals, order, and information. Scientific discoveries related to chaos, complexity, and emergence are given particular attention as this highlights the necessity to transition from a parts-based view to systems-based view.

Part III – Applications: Finally, we conclude with practical examples and methods for applying systems science to modern disciplines. Many of the environmental, social, and economical problems facing our world can only be addressed by thinking in terms of systems. Systems thinking provides an array of useful tools to create sustainable solutions and effectively intervene within our highly interdependent and complex world.

Systems science supports a new worldview of connection and complexity. The goal of this book is to catalyze that shift of thinking in you and society at large.

DAVID SHUGAR

PART I - FOUNDATIONS

Chapter 1 Systems

Example 1.1
Relationships

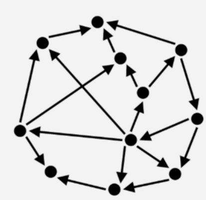
John Muir wrote *"When we try to pick out anything by itself, we find it hitched to everything else in the Universe."* This book uncovers the science of our connected and complex world.

Systems science presents a set of revolutionary ideas that prioritize viewing the world in terms of complex networks of relationships. Open and interconnected systems can generate chaotic patterns and emergent behaviors that are impossible to reduce to finite algorithms. Complex systems challenge the parts-based view that the whole can be efficiently predicted by the rules of the components. Instead of disjointed parts that sum in a linear fashion, systems science studies nature as a rich tapestry of nonlinear relationships. Systems science presents a new way to look at the world based on interdependence, complex patterns, and the connectivity between society and nature.

Studying systems provides a unifying approach to understand relations between elements for any given scenario, be it logical, physical, biological, sociological, or beyond. Examples of systems are included in Figure 1-1, each with particular elements, relations, and domains of applicability. Systems science works to establish common frameworks by which any model is proposed, taking a meta-view to study how models themselves are established. Scientific methods can then be used to assess the relevance of a given abstract model in understanding patterns in nature, with the goal of using models with increased accuracy and comprehensiveness.

Example 1.2
System Basics

A system can be drawn as elements (nodes) that follow certain relations (arrows). Systems can model abstract or natural patterns.



$$\begin{Bmatrix} \textit{System of} \\ \textit{Logic} \end{Bmatrix} \quad \begin{Bmatrix} \textit{System of} \\ \textit{Geometry} \end{Bmatrix} \quad \begin{Bmatrix} \textit{System of} \\ \textit{Atoms} \end{Bmatrix} \quad \begin{Bmatrix} \textit{System of} \\ \textit{Cells} \end{Bmatrix} \quad \begin{Bmatrix} \textit{System of} \\ \textit{Humans} \end{Bmatrix}$$

Figure 1-1 Examples of Systems

A Systems View

A motivational reason for a systems-based view is that the properties of the whole can be different than the sum of its parts. A parts-based view prioritizes isolated components where the property of the whole is a linear sum of its parts (e.g. total mass). However, in nonlinear systems with interrelated parts (e.g. trajectories of gravitation bodies), the whole can have emergent properties that are different than joining each part in isolation. The emergent effects of a whole can always, in principle, be constructed from joining the models of its parts, yet these emergent effects can be radically different. While linear models are typically easy to calculate, nonlinear models often generate chaos, complexity, and can be impossible to predict. A systems-based view studies nature through networks of simple to complex relations, rather than just considering disjointed, linear, and predictable pieces.

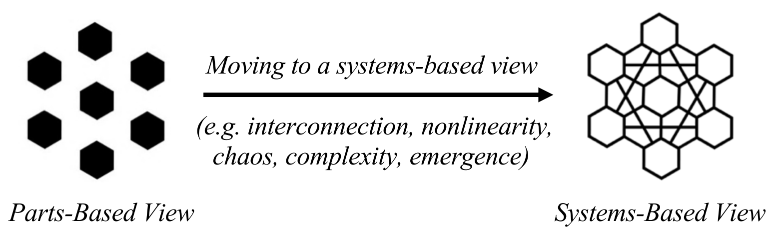


Figure 1-2 Parts-Based vs. Systems-Based View

Even systems with a few interrelated parts can lead to chaotic and irreducible results that cannot be solved with a finite number of steps. For example, the trajectories of two objects attracted by gravity can be quickly calculated for all cases, but there exists no general solution for the three-body case.[1] Most three-body trajectories, such as the one displayed in Figure 1-3, create chaotic orbits. Questions about the system, like if a given orbit will repeat, cannot be tested in a quick calculation prior to running a computer simulation that is subjected to timescale and resolution limits.

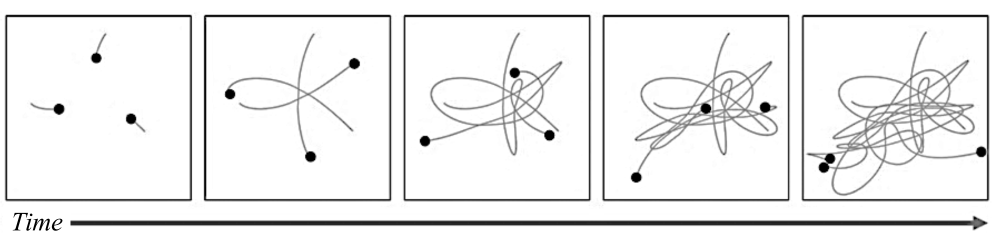


Figure 1-3 Chaos of Three Bodies in Gravity

Systems theory is not just about considering the behavior of a given model, but studying the process for how any given model can be established. A formal model consists of symbolic elements that follow initial rules, called axioms, that can used to prove results, called theorems. Some models, like a pendulum, create simple results that can be easily predicted, while others, like a double pendulum, create chaotic results that cannot be accurately predicted for long time scales. Systems theory studies how models themselves are created in any given field of study, which models are simple versus complex, and the connections between lower-level models to higher-level models of emergent behaviors.

Systems can be characterized by numerous measures and states. One distinguishing factor is if a system's boundary is closed or open to energy, matter, or information. Another example of a system state is thermodynamic entropy, which is a measure of disorder, energy dispersal, and the ability to do useful work. Yet another state is homeostasis, which occurs when a living system maintains a relative equilibrium between its interrelated biochemical reactions. Studying system measures and states, across different disciplines and emergent patterns, is foundational to systems science.

A critical measure of a system is complexity, or the relative difficulty of predicting future scenarios. One driver of complexity is nonlinear dynamics, which occurs when the total relation is not a linear sum of the pieces. Nonlinear systems can lead to chaotic patterns, unpredictability, and the butterfly effect, where small changes to initial conditions make large impacts over time. Complexity can also arise in self-organizing collections, such as bird flocks, schools of fish, and traffic jams. The higher-level patterns of these complex systems are often impossible to efficiently predict, even when the lower-level component rules are fully known.

In a more applied context, a systems view provides insights for how to enact effective change and create systemic solutions. A systems view is critical to addressing sustainability and socioeconomical problems with many interconnecting factors, from climate change to healthcare. Systems science presents a new paradigm, or underlying worldview, that nature and society must be understood as complex networks of connections, and that a relations-centric approach must be used to make effective change.

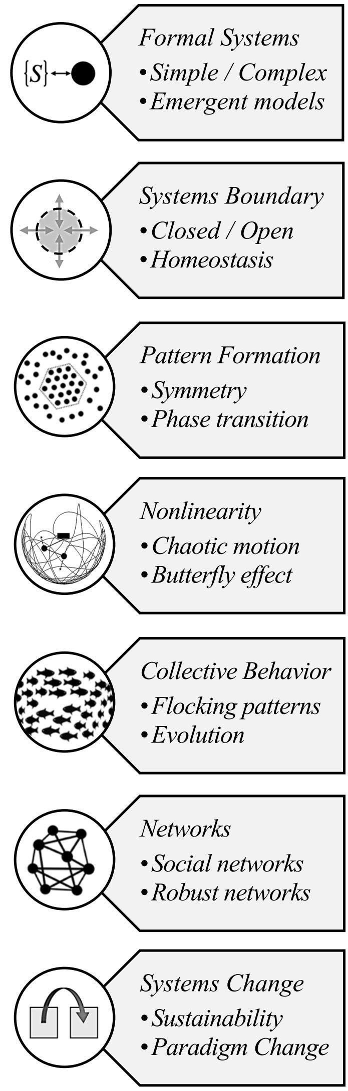


Figure 1-4 Topics of Systems Science

System Fundamentals

A system is a collection of parts that follow certain relations. In the solar system, individual planetary bodies are governed by the relation of gravitational force, among other forces. In political systems, the elements are human social agents, which relate through interpersonal activities. A system can also be an abstract collection of concepts, symbols and relational rules. Abstract systems, such as logic, math, geometry, and linguistics, provide foundational tools to analyze and model the systems in nature.

A pioneering figure in the concept of a general system was Ludwig von Bertalanffy, who first formulated the concept orally in the 1930s and later through publications.[1] Bertalanffy typically described systems as collections of objects that follow relationships modeled by differential equations, which describe how variables change over time. Bertalanffy also emphasized systems theory pivotal role in the unity of science, which is the notion that all the sciences should, in principle, be fully consistent and form a unified whole. The general study of systems provides a unifying approach to scientific knowledge and tools to communicate across disciplines.

A more formal definition of systems was introduced in the 1960s by Mihajlo Mesarovic, who defined a system as a set of relations between object elements.[2] This definition is inclusive of mathematical equations, but it is more powerful because relations can also include logical expressions. Mesarovic's definition was further developed by George Klir and others to define a general system to be an ordered pair of elements E and relations R, written as $S = (E, R)$. Other formal definitions of system have also been proposed that provide more details on different levels and complexity measures.[3]

This book will utilize the definition of a system as a set of elements E that follow a set of relations R, written $S = (E, R)$. These elements can be anything, like atoms, chemicals, organisms, social agents, or abstract symbols of a language, which follow particular relations that govern the behavior. For ease of expression, this book will often only denote the relevant governing rules rather than including all the elements and relations. For example, a system that contains and follows the rule $\{x = 0\}$ will be written as $S : \{x = 0\}$, with *":"* meaning *"such that"*, as shown in Figure 1-5.

$$S = (E, R) \qquad System = (Elements, Relations) \qquad S : \{Governing\ equations\}$$

Figure 1-5 Definition of a System

General System: A general formal system can take on any elements and relations relevant for a given model, such as an atomic system, a solar system, an ecosystem, and so forth. While traditional disciplines typically study one type of system in isolation, systems science takes an interdisciplinary approach to study all kinds of systems and the connections between systems. Commonly studied systems across disciplines, with their elements and relations, are shown in Figure 1-6.

System: *S*	Element: *E - (Types)*	Relations: *R*
Logic	*Proposition - (True/False)*	*Axioms*
Language	*Letter - (a, b, c, d, e, f, g, ...)*	*Grammar*
Computer	*Bit - (1 or 0)*	*Calculation rules*
Atom	*Quark - (Up/Down, Colors)*	*Strong force*
Molecule	*Atom - (H, He, Li, etc.)*	*Electromagnetic*
DNA	*Nucleotides - (A, T, C, G)*	*Molecular bonds*
Cells	*Cellular Parts - (DNA, ...)*	*Intercellular*
Neural Network	*Neuron - (Firing or Off)*	*Electric signals*
Ecosystem	*Organism - (Prey, Predator)*	*Food web*
Society	*Social Agents - (Humans, ...)*	*Social relations*

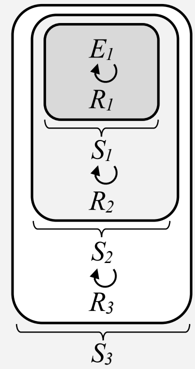
Figure 1-6 System Units Across Disciplines

An intriguing interdisciplinary result is that one model can be applied to surprisingly different real-world systems. For example, exponential growth equations that model bacteria populations can also model investment growth.[4] Bacterial growth and compounding investments are not driven by the same mechanisms, but can be analyzed in similar ways because they follow similar growth patterns. These commonalities are due to the fact that "formal" models, like math and logic, are true by virtue of their form irrespective of the particular content. Math does not care what a variable, like *x,* is referring to in a given equation, like $x_1 = 1.10(x_0)$. While formal systems are indifferent to what they represent, the scientific method works to identify more useful models to understand nature.

An important concept is that an element's identity in a system depends on the relations that govern it. In order for a given variable, like *x,* to have meaning, it needs to be related to something else in the formal system. Once a given relation is defined in a system, like *x* is twice as large as *y,* then the various parts are given relevant meaning. Even the identity transformation, *x* is equal to *x*, is a type of relation. An element's identity depends on the relation of being equals to itself, $x = x$, and not equal to other elements, $x \neq (not\ x)$. At its core, formal systems are about the relations between things, not things themselves.

Example 1.3
Nesting Systems

The elements of a system can include other systems, and nest together.



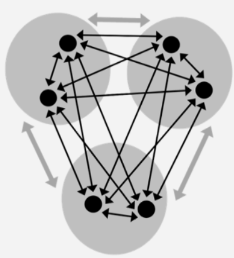
$S_1 = (E_1, R_1)$
$S_2 = (S_1, R_2)$
$S_3 = (S_2, R_3)$

A system S_2 can add relations R_2 on another system S_1. Only equivalent or emergent, rather than new, relations are allowed when each nested model interprets shared universal behavior.

Emergent Models: Emergent, higher-level, systems S_{Higher} model a subset (a part of the set) of the domain of scenarios from lower-level models S_{Lower}, following the mapping $limit(S_{Lower}) \rightarrow S_{Higher}$. A common way to limit a system is to only consider collective properties. For example, the higher-level model of temperature is based on the average energy in a collection of particles and does not model each particle's state. Many properties of natural systems, like phase, solidity, conductivity, and chemical reactions are emergent properties that only come about in large collections. Even more emergent properties come by limiting systems in other ways, such as setting variables, like friction, to zero as well as limiting the ranges of energies considered.

Example 1.4
Emergent Collections

In a joining limit, the lower-level system S_L models parts, while the higher-level system S_H models collections. In emergent models, S_H is only mapped one way from joining S_L and is unequal as detail is lost, $Join(S_L) \rightarrow S_H$. In equivalent models, S_H equals the joined S_L and can be mapped both ways, losing no detail, $Join(S_L) \leftrightarrow S_H$.



$S_L = (\bullet, \longleftrightarrow)$
$S_H = (\bullet, \longleftrightarrow)$

$limit(S_{Lower}) \rightarrow S_{Higher}$

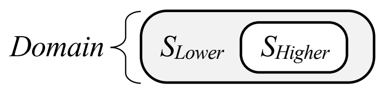


Figure 1-7 Emergence Definition

Emergence plays an essential role in bridging between disciplines. In principle, the lower-level model of physics maps to the higher-level model of biology, biology maps to psychology, and so on. While higher-level domains are always of subset of lower-level domains, many emergent models are not efficiently predictable by lower-level models. Higher-level theories are often found without being derived from lower-level counterparts. Systems science simultaneously develops theories addressing different domains, like multi-layered maps of a single reality.

Real Systems: The systems of nature, or real systems, is an idealized notion how nature itself functions. A given model can never be proven to truly describe nature and real systems are in general unknowable, however, scientific methods attempt to provide the best possible explanations. In current scientific theories of real systems, spacetime creates the background for the distribution of matter, energy, and information, which follows the principle of least action, conservation laws, and other relations. These models include an asymmetric dimension of time that began at the Big Bang and allow processes to be either reversible or irreversible. Real systems display a wide range of patterns across different sizes and complexity levels that are described by emergent models, such as physics, biology, and sociology. Some common patterns in modeling natural systems are introduced in the following section and expounded on in subsequent chapters.

Boundaries and Flux: Natural systems are often defined with boundaries, which may be open or closed to change. The symbol Δ represents the change of a quantity X from initial to final states, written $\Delta X = X_{final} - X_{initial}$. A closed system has no inputs and outputs over the boundary, and requires that conserved quantities are in equilibrium $\{\Delta X_{Conserved} = 0\}$. In contrast, an open system can have inputs and outputs over the boundary, which allows conserved quantities to have a non-zero change and be in flux $\{\Delta X_{Conserved} \neq 0\}$. In thermodynamics, "isolated systems" have no change in matter and energy, while "closed systems" have no change in matter, but are open to energy. The concept of closed versus open systems can apply to many different fields, from engineering to ecosystems.

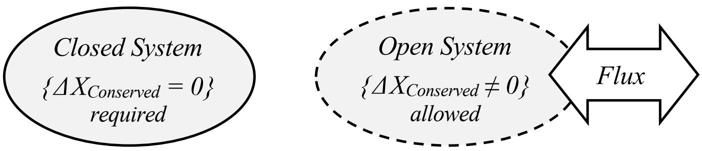
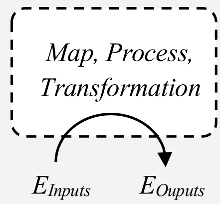


Figure 1-8 Closed vs. Open System

Example 1.5
Inputs to Outputs

Systems can express how inputs E_{Input} transform, or map → over a process, to outputs E_{Output}. The change is defined as $\Delta E = E_{Output} - E_{Input}$, showing quantities that are steady or differ over a process.



Symmetry and Fractals: Identifying repeating and symmetrical patterns can provide insight into a system's structure. Symmetry is defined as changes in a system that result in a state identical to the initial state. A symmetry follows the equation $\{X \rightarrow X\}$, where a given transformation or mapping →, like a rotation or movement, results in the same initial state X. A bilateral image is one that can be flipped along an axis of symmetry to produce an image indistinguishable from the original. Fractals are another type of symmetry that repeat over size scales, instead of rotational or translational movement. Symmetry can be observed in many natural patterns, from cubic crystal lattices to the fractal patterns of a Romanesco broccoli. Symmetrical patterns often arise in nature because these patterns frequently minimize energy and resources.

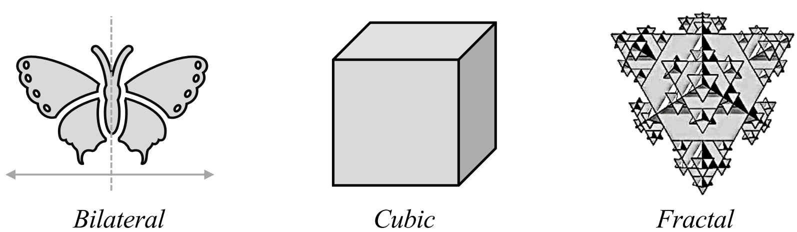


Figure 1-9 Symmetry in Systems

Entropy and Order: The interplay of order and disorder, measured by entropy, plays an essential role in systems. Entropy grows when there is a higher number of equally likely microstates that result in the same overall macrostate. Ordered systems have low entropy and disordered systems have high entropy. For example, Figure 1-10 graphs 100 boxes that are half white and half grey. There is only one state for all grey boxes to be in the center, which is considered highly ordered. In contrast, there are many possible states where grey and white boxes are intermixed among the interior and exterior regions. The most complex systems, like life, often arise in the transitional space between order and disorder.

Example 1.6
Arrow of Time

The assumption of a low entropy past, like the Big Bang, allows the entropy of the universe to increase over time, enabling the past to be distinguished from future states in a direction, or arrow. This occurs even though lower-level physics is time reversible, which means it looks the same forwards or backwards in time.

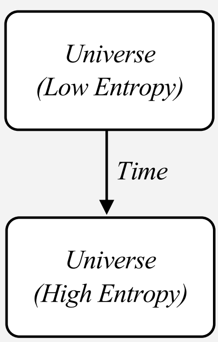


Figure 1-10 Entropy and Order in Systems

In thermodynamics, entropy relates to energy dispearsal and the decreased ability for a system to exert useful work. The second law of thermodynamics states that the entropy of an isolated system tends to increase, leading to energy dispersal and disorder over time. However, it is possible for a subsystem within an isolated system to receive energy to reduce entropy within a bounded region. Energetically open systems, like steam engines and metabolic reactions, can use external energy sources to power entropy-reducing processes that are able to increase order and organization.

Information Processes: Material and energetic patterns in spacetime can serve as markers, which store information that can be encoded or decoded via communication, computational, and informational systems. Information is defined through Shannon entropy in the units of bits, which relates to how much surprising information is provided by a message. Communication systems can be built to encode, decode, and transmit information over channels, like sending music over radio waves. Additionally, computers can be built to store, process, and manipulate information. Information also plays a critical role in biological systems, such as DNA and neural networks. Physical systems can either be influenced by matter and energy, or influenced by information marked by matter and energy, or some combination of the two.

Complexity and Irreducibility: A critical attribute of a system is the relative difficulty of predicting final states. Some algorithms, like multiplication, are easy to solve for large numbers. In contrast, problems optimal chess moves become increasingly difficult when scaling up in size. Many systems in nature, like weather patterns and biological processes, have difficult or impossible to solve algorithms when the number of components increases, requiring an unfeasible amount of computational resources to predict future states.

Algorithms describing complex systems often have no efficient means produce solutions. The halting problem, for example, considers a computer program that follows specific rules of manipulating symbols that may lead to a command to halt. Knowing the rules of the program does not tell you in advance if a program will, or will not, halt. The only way to test halting is to run the program, potentially indefinitely, with no knowledge if the program will ever stop. Alan Turing proved in 1936 that a general algorithm to solve the halting problem cannot exist.[5] The halting problem is undecidable, meaning a yes or no answer cannot be established ahead of time.

Example 1.7
Complex Systems

In a complex system, predictions can be impossible to reduce to a yes or no answer. Examples include:

- *Weather*
- *Ecosystems*
- *Economics*
- *Politics*
- *Society*

***Sustainability and Resilience*:** Systems theory provides a toolset both to model the connections of society and nature, and to design sustainable solutions. System dynamics, for example, models resource stocks, flow rates, and feedback. A sustainable system occurs when the replenishment rate equals the extraction rate, as shown in Figure 1-11, written as *{ΔReservoirs = 0}*. Resource reservoirs must be used at a rate below replenishment in order for humanity to sustainably live on Earth and preserve resources for future generations. Systems thinking can also be used to identify ways to reduce resource use across sectors like energy, waste, and buildings, as well as create resilient social networks that adapt to unforeseen volatility.

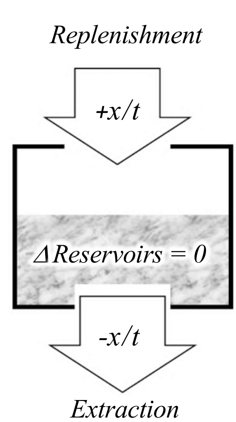


Figure 1-11
Sustainable Reserve

***Transforming Systems*:** Transformation is the change of a system itself over time, written as $S_{Initial} \neq S_{Final}$. Humans are continually changing our scientific and socioeconomic systems. Facilitating change is critical to creating large-scale impact and effective solutions. Systems thinking provides essential tools for enacting effective change and creating processes that can adapt over time.

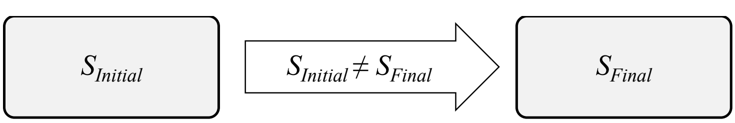


Figure 1-12 Transformational Systems

Foundational to Applied Systems

Systems can be categorized along a spectrum of foundational to applied systems. Foundational systems pose axioms, or rules, of a given model. Theoretical systems then use deductive reasoning to prove if a given conclusion is true following a set of axioms. For example, there are many theorems that are provable with the axioms of Euclidean geometry. Methodological systems perform tests to gain further results, such as physically measuring geometric ratios rather than finding exact proofs. Applied systems apply functional models to real-world scenarios, like applying geometry to building construction. Methodological and applied systems also use inductive reasoning, which extrapolates observations into larger conclusions as well as abductive reasoning, which considers which predictions work the best. However, as expounded by the philosopher David Hume, just because an event has occurred numerous times does not mean that it is guaranteed to occur again.[6] Methodological and applied systems are not guaranteed to be completely valid, just practical. The goals of foundational to applied systems are summarized in Figure 1-13.

Foundational Systems	*Pose a model that follows specific axioms, or rules*
Theoretical Systems	*Extrapolate from axioms to other provable models*
Methodological Systems	*Utilize tests and approximations to inform models*
Applied Systems	*Models for implementation in real-world systems*

Figure 1-13 Foundational to Applied Systems

Foundational and theoretical systems include abstract fields like set theory, logic, and math. Fields like physics, chemistry and biology build-off foundational systems to specify theories of nature. Science attempts to model real systems as accurately as possible by comparing models with large empirical datasets. Another way to distinguish useful scientific theories is through a model's ability to be falsified. Being able to be tested to be true or false is essential to provide additional benefit to explaining nature. Fields like sociology, economics, and politics study a variety of more applied systems in our world. Methodological and applied systems do not always have a strict or consistent set of axioms, but rather work to find what models serve the best functional purposes. Examples of foundational to applied system are summarized in Figure 1-14.

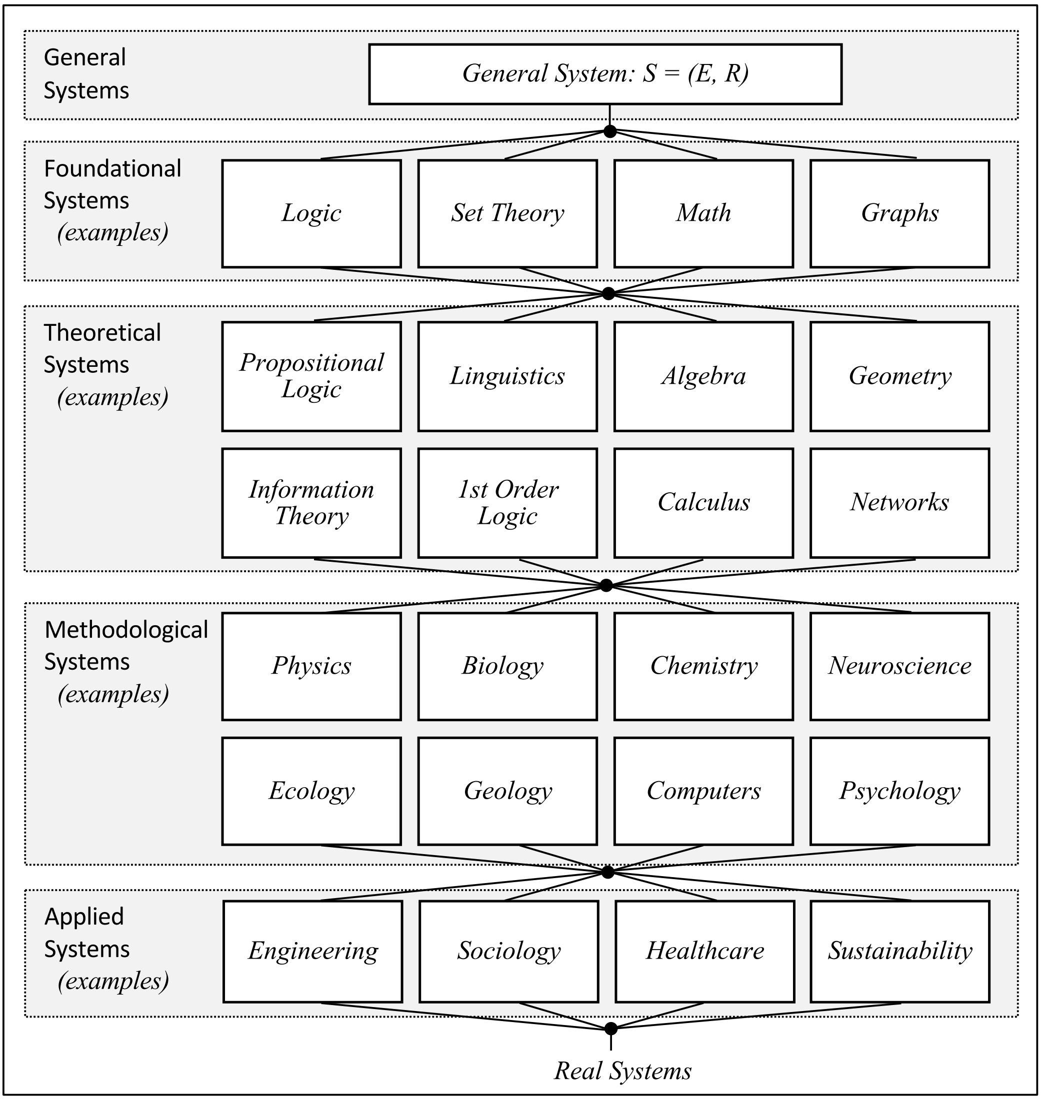
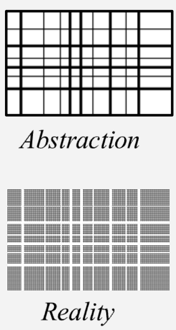


Figure 1-14 Foundational to Applied Examples

All methodological and applied systems about nature should, in principle, be consistent and form a unified whole. This unity follows the assumption that science identifies universally valid principles of one reality that are symmetrical to any place, time, and scales of size. While models of nature may be split into different types and applicable domains, nature itself is not delineated between physics, biology, and other fields. Even though this unity is commonly assumed, disciplines often become highly specialized and siloed. System theory supports the unity of science by providing unifying terminologies and methodologies that crosses all disciplines of logic and science.

Example 1.8
Map vs. Reality

A map of reality should not be thought of as the landscape itself. Similarly, informal or formal systems should not be considered real systems, whose actual properties are unknowable.



Formal systems, like logic and math, may attempt to model nature, yet there is no test to confirm that a model is how reality functions. This concept is summarized by statistician George Box's phrase, "All models are wrong, but some are useful."[7] Real systems are similar to a black box, where experiments can be tested to determine what may be inside the box, but the exact internal mechanisms cannot be known. At the same time, formal systems arise within and are limited by real systems. For example, physical markers (e.g. written symbols, neural activity, computers) can symbolize a formal system that provides a model beyond the universe, however it is impossible for physical markers themselves to violate the properties of real systems. The modeler is always part of the reality being modeled and there can never be a truly objective perspective of nature.

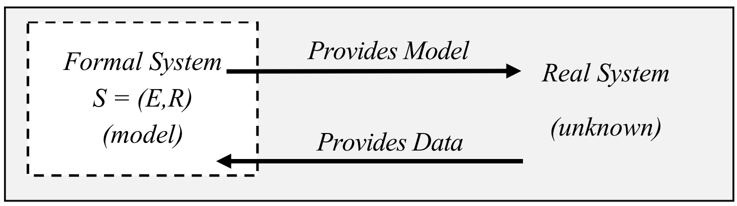


Figure 1-15 Formal and Real Systems

There is a difference between logical versus real-world validity. Formal logical and mathematical systems are true or false based only on the form of the argument, not on real-world data. For example, the transitive argument—if $a = b$ and $b = c$, then $a = c$—is always true, regardless of the specific content. Even the argument, "If you are a human and all humans are immortal, then you are immortal," is a valid formal statement. Obviously, humans do not live forever, so this statement would not be expected to be true based on the background theories of biology. Methodological and applied systems, like physics and biology, work to understand if evidence supports if the content within a given formal model is expected to be valid.

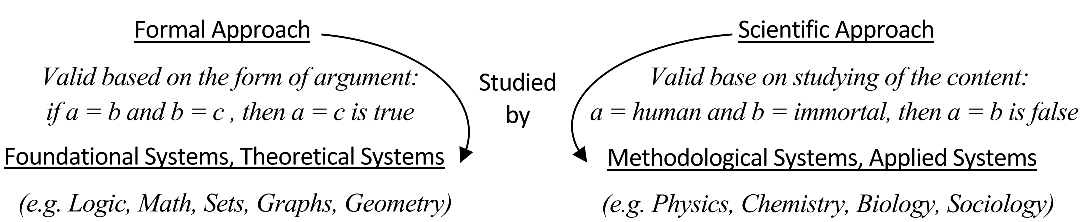


Figure 1-16 Formal and Scientific Approaches

Systems science lies at the intersections of formal systems and real systems. At the formal level, systems science works to improve theoretical concepts and representations. On the applied side, systems science works to develop toolsets, practices and methodologies that are backed by real-world results. At the intersection of formal models and the real-world, systems science studies fundamental principles useful for natural sciences, such as boundaries, flux, entropy, complexity, and emergence, as shown in Figure 1-17. System fundamentals supports the unity of science by providing overarching principles of systems that allows communication across all disciplines.

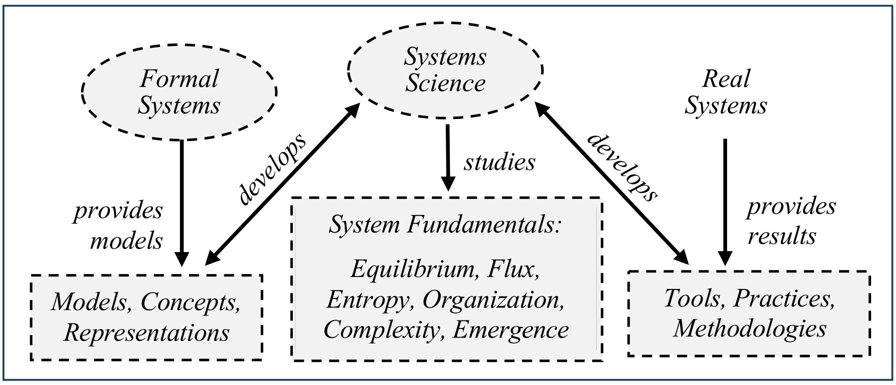
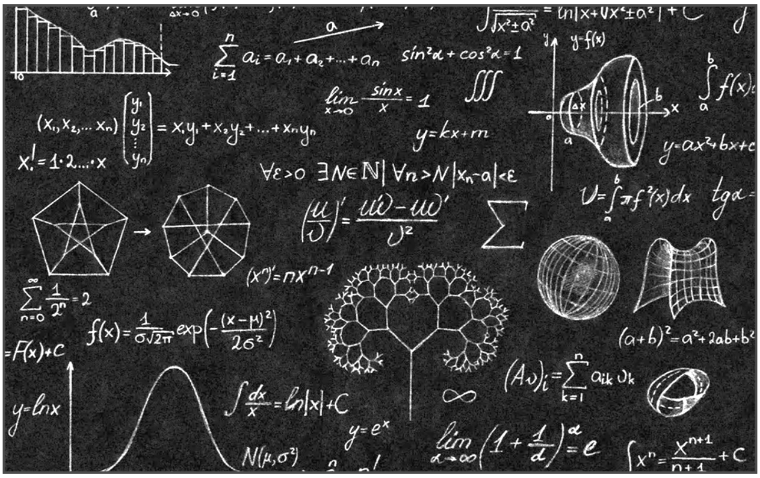


Figure 1-17 Systems Science Across Theory and Practice

Summary

Systems science provides a unifying view to study the world, based on connection and complexity. A system, defined as a set of elements and relations, can apply to different scenarios. Foundational and theoretical systems create abstract formal models that can be used in methodological and applied systems to study real-world systems. Systems across disciplines have reoccurring patterns, such as equilibrium, flux, symmetry, organization, complexity, sustainability, and transformation, which will be elaborated on in subsequent chapters. Additionally, the concept of emergent patterns is critical to understand the distinctions between lower-level and high-level models of nature. Systems theory takes an interdisciplinary educational approach, where multiple models—like physics to biology—are considered together in parallel and are ultimately fully consistent and unified descriptions of nature.

Chapter 2 Formalization



Example 2.1
General System

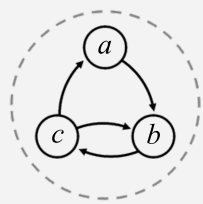
A general system can represent any given formal model, like logic, math, graphs, networks, and computer processess.

This chapter will provide a generalized language to define a system, regardless of what kind of elements and relations are being considered. A general system definition provides insights for how models themselves are established, providing a singular approach that can be applied to patterns that span all sizes, time scales, and complexities. Systems science is in large part a metatheory, which studies how theories themselves are developed. Contrary to traditional science, which primarily studies specific models, systems theory develops unifying methods that apply to any kind of model.

Set theory can be used to bring further clarity to defining a system. A set is defined as a collection of members elements *E*. Going beyond this, a system is a collection of object elements *E* that also follow relations *R,* as shown in Figure 2-1. For example, a set of books would just include an unorganized collection of books, while a system of books can include additional relations, such as alphabetical order, author name, and categories that systemize the collection. A system can also simplify into a set when the relations are empty or equals *E*. The elements and relations of a general system can be further specified to apply to certain logical, mathematical, or formal models. These models can then be used to study real-world phenomena in physical, biological, and sociological systems.

Example 2.2
Graphing Systems

The system *S* below has three elements *a, b, c,* as well as the relation drawn with arrows.



In set notation:

E = {a, b, c}
R = {(a,b), (b,c), (c, a), (c, b)}

$$S = (E, R)$$

Figure 2-1 Equation for a System

Elements and Relations

> **Example 2.3**
> Relation Tables
>
> Relations can be written as a tables, such as a collection of people that are part of different social groups.
>
> | | |
> |---|---|
> | *Ava* | *Student* |
> | *Ben* | *Student* |
> | *Ben* | *Parent* |
> | *Bob* | *Parent* |
> | *Bob* | *Teacher* |
> | *Ted* | *Student* |
>
> This relation can be written as pairs:
>
> $$R = \left\{ \begin{array}{c} (Ava, Student) \\ (Ben, Student) \\ (Ben, Parent) \\ \ldots, \ \ldots \end{array} \right\}$$
>
> Multiple tables can be linked to make a relational database.

Systems can be defined with set theory as a collection of elements E and relations R, written $S = (E, R)$. The elements of a system could be anything, like three arbitrary members $E = \{a, b, c\}$, the group of two people $E = \{Joe, Jane\}$, a list of numbers $E = \{2, 3, 5, 7\}$, or logical propositions, $E = \{p, q\}$. The elements could continue infinitely, like the natural numbers $E = \{0, 1, 2, 3, \ldots\}$, contain other sets $E = \{\{a_x, a_y\}, \{b_x, b_y\}\}$, or be other types of mathematical objects and logical classes.

A system is more general than a set because there exist additional relations R that add a structure of rules to the elements. For example, a system could contain the variables x and y that follows the relation that $R = \{x + y = 10\}$. Numerous relations, or rules, can simultaneously apply to elements, such x plus y equals ten, $R_1 = \{x + y = 10\}$ and x is greater than y, $R_2 = \{x > y\}$. Multiple mathematical rules form a system of equations, which may be solvable or unsolvable. More generally beyond math, a system is any collection of elements that follow any set of relational rules.

A relation R describes how one set maps to another set and can be represented as an ordered list, or drawn as arrows. For example, consider the two sets of $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$, where a is a member of the set A and b is a member of B. The relation of equality $a = b$ are the pairs of (a, b) that are equal in both sets, which has the solutions $\{(1, 1), (2, 2)\}$. The relation of greater than, $a > b$, has the solutions with the pairs of $\{(2, 1), (3, 1), (3, 2)\}$, meaning $2 > 1$, $3 > 1$, and so on. The relation of $a < b$ has the solutions $\{(1, 2), (1, 4), (2, 4), (3, 4)\}$. These different relations are drawn also below in Figure 2-2. A system can have any number of relations, which are able to provide insights to the patterns of how elements relate to one another.

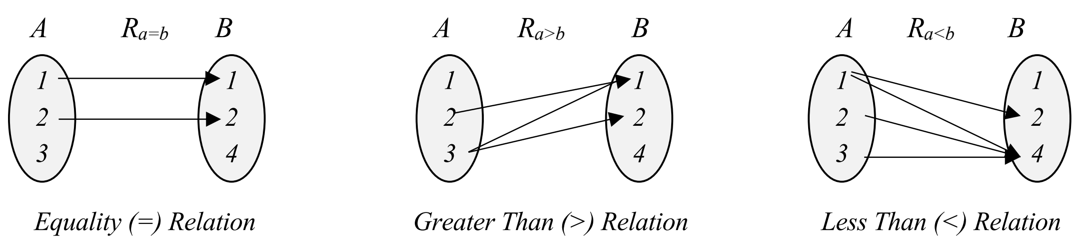


Figure 2-2 Defining Relation Sets

Relations can represent mathematical functions, which map specific inputs to certain outputs as pairs (*input, output*). For example, the relation pairs $R = \{(1, 1), (2, 4), (3, 9), (4, 16) ...\}$ follows the pattern that an input of x outputs x^2, such as $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, and so forth. This relation can also be represented as the function $f(x) = x^2$, where the input of x will lead to the output equal to x^2. Even more complicated functions can be written in pairs, such as $f(x) = x^3 + x$, written $R = \{(1, 2), (2, 10), (3, 30), ...\}$. Relations can also include multiple variables such as $f(x, y) = x + y$, which has two inputs of x and y, and a single output, written as $R = \{((1, 2), 3), ((1, 3), 4), ...\}$ expressing $1 + 2 = 3$ and so forth. Other equations for systems, like equilibrium or flux, can be expressed as relations as well.

Relations can be defined through the Cartesian product, which contains all ordered lists of elements from multiple sets. For example, if $A = \{a_1, a_2\}$ and $B = \{b_1, b_2\}$ then the Cartesian product $A \times B$ equals $\{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2)\}$, or all pairs (a, b) where a belongs to A and b belongs to B. A binary relation on the two sets A and B is equal to a given subset $\subseteq$ of the Cartesian product, written $R_{Binary} \subseteq A \times B$.[8] Relations, like $a = b$, $a > b$, or $a < b$, are all certain subsets of $A \times B$. The Cartesian product can be extended to more sets, like the three sets $A \times B \times C$ that define all possible triplets (a, b, c).

Relations can take different numbers of elements, determining the "arity". A unary relation has a single element, like (a), while a binary relation considers two elements, like (a, b). A ternary relation represents the relations between three variables, such as (a, b, c). For example, a ternary relation could be that a thinks that b and c are friends, requiring three variables. A n-ary relation uses n number of variables $(e_1, e_2, ... e_n)$ and is the subset of $R \subseteq E_1 \times E_2 ... E_n$ times.

A general system can be formally defined as a pair of elements that follow relations, $S = (E, R)$. Each relation $R = \{R_1, R_2, ... R_r\}$ is a subset of the Cartesian product of elements to the n-th order, following $E \times E \times E \times ... E_n = E^n$. For example, with the elements of two sets $E = \{E_1, E_2\}$ (e.g. inputs and outputs) and $n = 2$, the system's relations will be binary, which follows $R \subseteq E \times E$, or $R \subseteq \{E_1, E_2\} \times \{E_1, E_2\}$. This encompasses the binary relations between the two sets and to themselves, written $R \subseteq \{(E_1, E_1), (E_1, E_2), (E_2, E_1), (E_2, E_2)\}$.

$$E = \{1, 2, 3\}$$

$$E \times E = \left\{ \begin{matrix} (1,1), (1,2), (1,3) \\ (2,1), (2,2), (2,3) \\ (3,1), (3,2), (3,3) \end{matrix} \right\}$$

$$E \times E \times E = \left\{ \begin{matrix} (1,1,1), (1,1,2), (1,1,3) \\ (1,2,1), (1,2,2), (1,2,3) \\ (1,3,1), (1,3,2), (1,3,3) \\ (2,1,1), (2,1,2), (2,1,3) \\ (2,2,1), (2,2,2), (2,2,3) \\ (2,3,1), (2,3,2), (2,3,3) \\ (3,1,1), (3,1,2), (3,1,3) \\ (3,2,1), (3,2,2), (3,2,3) \\ (3,3,1), (3,3,2), (3,3,3) \end{matrix} \right\}$$

Figure 2-3 Cartesian Product of Sets

$$S = (E, R) \qquad System = (Elements, Relations)$$

$$E = \{E_1, E_2, ... E_e\} \quad R = \{R_1, R_2, ... R_r\} \quad R_r \subseteq E^n \quad n = Relation\ order$$

Figure 2-4 General System Definition

Operator Order

Formal systems often introduce operators, a type of relation that maps an input into a single output value. Operators are less general than relations, as they only allow one-to-one mappings of inputs to outputs. Like relations, operators have different "arity", or order, depending on the number of input elements. A unary operator relates one element to an output. For example, in math the negative symbol is a unary operator that can be put in front of a single element, leading to the function $f(x) = -x$ that describes how an input of x outputs $-x$. A more general way to define a unary operator is with a function from a single input to an output, written $f: E_1 \rightarrow E_{Out}$, or as the pair (E_1, E_{Out}).

A binary operator connects two elements. An example of a binary operator is the plus symbol that adds two terms $f(x, y) = x + y$. Binary operators are defined by the function $f: E_1 \times E_2 \rightarrow E_{Out}$, and can be represented as a relation with an ordered triple (E_1, E_2, E_{Out}). In general, a n-ary operator can be expressed by $(n+1)$-ary relations. The relations are one term longer to account for the output E_{Out}. There are also ternary and n-ary operators, which can be expressed as the combination of multiple binary operators.[9] Many types of operators are used in formal systems to provide practical ways to deduce results, some of which are shown in Figure 2-5.

Relation Order	Operator Order	Set Theory $E=\{members\}$	Mathematics $E=\{variables\}$	Logic $E=\{propositions\}$
$n = 1$ (E_{out})	*Nullary Operator* $f: \{\} \rightarrow E_{Out}$	a *Members*	C *Constant*	⊤ *Truth* ⊥ *Falsity*
$n = 2$ (E_1, E_{out})	*Unary Operator* $f: E_1 \rightarrow E_{Out}$	C *Compliment* | *Cardinality* *P* *Power set*	– *Negative* ! *Factorial* *log* *Logarithm*	¬ *Negation* *p* *Identity* ∃ *There exists*
$n = 3$ (E_1, E_2, E_{out})	*Binary Operator* $f: E_1 \times E_2 \rightarrow E_{Out}$	| *Such that* ∈ *Member of* ∩ *Intersection*	= *Equals* + *Addition* · *Multiply*	∨ *Or* ∧ *And* → *Implies*
$n = 4$ (E_1, E_2, E_3, E_{out})	*Ternary Operator* $f: E_1 \times E_2 \times E_3 \rightarrow E_{Out}$	*Pairing operator for three members (a, b, c)*	*Triple cross product of vectors* $A \times B \times C$	*If p and q are true, then r is true* $(p \land q \rightarrow r)$
	…	…	…	…
$(n + 1)$ $(E, \ldots E_n, E_{out})$	*n-ary Operators* $f: E_1 \times \ldots E_n \rightarrow E_{Out}$	*Order list of n* $(a_1, (a_2, (a_3, .. a_n)$	∑ *Sum of n* ∫ *Integral*	*If* a_1 *and* a_2 … *and* a_{n-1}, *then* a_n

Figure 2-5 Operators in Formal Systems

Operators are commonly used in logic. The unary truth operator maps an input proposition *p* of true *T* or false *F* to an output result, written as $p \rightarrow output$ or the pair (*p*, *output*). In this case, there are four unique ways to match each input with a single output. These four variations represent the four unary truth operators. When the output is equal to the proposition's truth value, *{(T, T), (F, F)}*, it represents an identity of *p;* when the truth value is opposite, it is called the negation ¬. A tautology ⊤ occurs when the output of *p* is always true, like "$x = y$ or $x \neq y$", and has the pairs *{(T, T), (F, T)}*. A contraction, or falsity ⊥, occurs when *p* is never true, like "$x = y$ and $x \neq y$". The relation arrow diagrams in Figure 2-6 represents the unary logic operators.

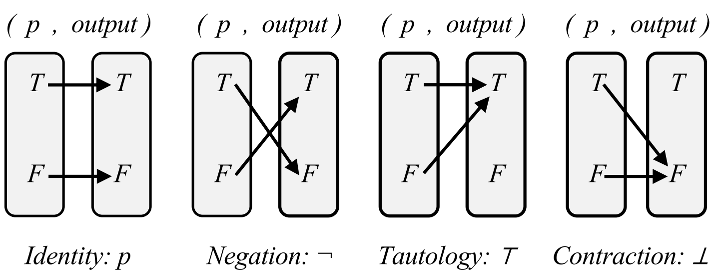


Figure 2-6 Unary Logic Operators

Logic heavily utilizes binary operators, like *and, or,* and *implies,* that map two input propositions of *p* and *q* to an output, written $f\colon E_1 \times E_2 \rightarrow E_{Out}$. The product *{T, F}*×*{T, F}* has four pairs *{(T, T), (T, F), (F, T), (F, F)}*. The binary operators describe how these four values can map to unique values of truth or falseness. The binary operator *and,* written as ∧, means both *p* and *q* must be true for the output to true, written *(T, T)* → *(T)*. There are 16 unique binary operators, including *or, implies*, and others that map (*p, q)* to an output, four of which are displayed in Figure 2-7.

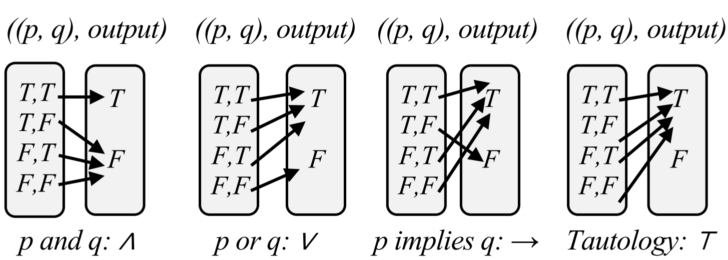
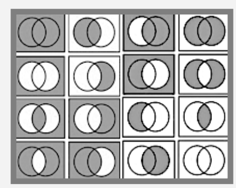


Figure 2-7 Binary Logic Operators

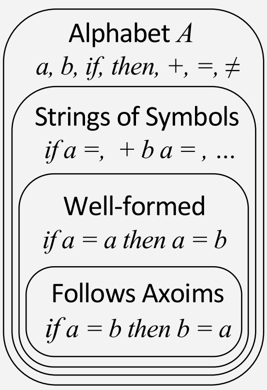
Example 2.4
Truth Diagram

All 16 binary logical operators can be displayed as Venn diagrams of two circles, representing two propositions (inside circle) and their negations (outside circles) filled for in truth or left blank for false.



Example 2.5 Symbols and Formal Systems

Formal systems are made of symbols from an alphabet A, and relational rules between symbols. A subset of all possible strings of symbols are well-formed grammatically, and a subset of well-formed statements follow the axiomatic rules of a langauge.



$S_{Language} \subseteq A^*$

$A^* = A \times A \times \ldots A_\infty$

Formal Systems

One of the most basic formal systems is a formal language, which has the elements of an alphabet A and alphabet lists (called strings) X that follow certain relations R. A formal language can be defined as $S_{Language} = (A, X, R)$, which arises from the general system definition $S = (E, R)$ when $E = (A, X)$. For example, the alphabet could be Latin script $A = \{a, b, c, \ldots\}$ and one string could be a story, $X = \{Once\ upon\ a\ time\ \ldots\}$. The relations R represent the rules of the alphabet used in the language, such as grammar rules or other spelling patterns that apply.

A useful tool to study formal languages is to take the set of all possible arrangements of the alphabet, written as A^*. This produces an endless list equal to the infinite Cartesian product, written $A^* = A \times A \ldots \times A_\infty$. For $A = \{a, b\}$, the term A^* would include $\{\{\}, (a), (b), (a, a), (a, b), (b, a), (b, b), \ldots\}$. For a given alphabet, A^* represents all possible combinations of letters and contains every expressible message, story, or theory. Most of these lists are not well-formed, but a certain subset of combinations forms words and grammar that are part of the language, written $S_{Language} \subseteq A^*$.[10] When the entire behavior can be defined by one relation, a general system also equals $S \subseteq E^*$.

Another formal system is an automaton, which describes the manipulation of symbols, like a computer. An automata with finite inputs is defined as $S_{Automaton} = (A, Q, Q_0, Q_f, R)$.[11] Similar to a formal language, an automaton contains an alphabet of symbols A, but also a set of states, $Q = \{Q_0, Q_1, Q_2, \ldots\}$ that could represent things like on, off, or specific values. This system also includes an initial state Q_0 as well as the set of allowed final state(s) Q_f, which enables the program to stop. To determine how the system changes and manipulates the symbols, a relation R is introduced called the state-transition function, which maps the states and symbols to a new formulation $R : Q \times A \rightarrow Q$.

The alphabet and states of an automata can have both internal inputs and external outputs. This is seen in computers that relate many internal states of 1 or 0 to external keyboards and visual displays. This is accomplished by having both an input alphabet A_i and output alphabet A_o, each with their own mapping relation, following $R_i : Q \times A_i \rightarrow Q$, and $R_o : Q \times A_o \rightarrow Q$. Other automata introduce other measures, such as a separate tape that can serve as a memory storage, and expanding sets to non-finite values.

Formal systems of logic provide a means to assess truth statements and deduce conclusions for a formal language. The elements in a logical system include propositions (statements that can either be true or false) $P = \{p_1, p_2, p_3, \ldots\}$. A logical system features various kinds of relations, including a set of operators O that connect propositions, like $p \wedge q$ *(p and q)*, $p \vee q$ *(p or q)*, or $p \rightarrow q$ *(p implies q)*. Logical systems also have the relations of inference rules I and axioms Z, which are the valid rules for assessing truth. Together, a logical system can be defined as the ordered list $S_{Logic} = (P, O, I, Z)$.

> **Example 2.6** Laws of Thought
>
> Three classical laws of logic are:
>
> *Identity*
> *For all p, it is true that p equals p.*
>
> *Non-contradiction*
> *Both p and ¬p can not be true.*
>
> *Excluded Middle*
> *Either p or ¬p is true, not both.*

Logical systems can combine multiple propositions to reach a conclusion through the rules of inference. For example, if p is true and $p \rightarrow q$, then q is true. Also, if $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$. The rules of inference in proposition logic are included in Figure 2-8 and use the three dotted symbol ∴ to note when conclusions are made. Steps taken to reach a logical conclusion can also be expressed in truth tables or with Boolean algebra, which assign the value *1* to true and *0* to false. The underlying set of inference rules I and the axioms Z can change depending on the type of logical system being considered. First-order logic introduces new axioms and considers propositions with quantifiers, like *for all x,* or *there exist x*. There is also modal logic, which studies propositions that can be possibly true or not, and other kinds of logical systems that extend beyond traditional logic.

Premise 1	$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$	$p \vee q$	$p \vee q$	p		
Premise 2	p	$\neg p$	$p \rightarrow r$	$\neg p$	$\neg p \vee r$	q	p	$p \wedge q$
∴ Conclusion	$\therefore q$	$\therefore \neg q$	$\therefore p \rightarrow r$	$\therefore q$	$\therefore q \vee r$	$\therefore p \wedge q$	$\therefore p \vee q$	$\therefore p$

Figure 2-8 Rules of Inference in Propositional Logic

Equivalence is an essential relation to transform statements into alternative, equal, forms to find solutions. An equivalence relation has reflexivity ($p = p$), symmetry (if $p = q$, then $q = p$) and transitivity (if $p = q$ and $q = r$, then $p = r$).[12] In category theory, which studies objects and their mappings, called "morphisms" (relations symbolized by →), objects like x and y are "isomorphic" and have equivalent underlying structures when there is a morphism from both $x \rightarrow y$ and $y \rightarrow x$. It is essential that an isomorphism goes both ways to ensure the structures of the of objects are equivalent and have a one-to-one mapping to each other. Transforming logical statements and objects into new equivalent forms is an indispensable tool to prove if a premise is true, false, undecidable, or leads to contradictory results.

Another formal system is a mathematical system, defined as $S_{Math} = (E, O, Z)$, which has the elements of mathematical objects and the relations of operators and axioms. Arithmetic considers elements, like the natural numbers $E = \{0, 1, 2, 3, \ldots\}$, and operators like addition and multiplication, $O = \{+, -, \cdot, \div, \ldots\}$. These binary operators relate two elements and are a $n = 3$ relation, $(x, y, output)$. Different mathematical systems introduce a variety of other operators and axioms Z. For example, the axioms of algebraic fields include identity, inverses, commutativity, and the other rules in Figure 2-9.[13] Following these basic rules, mathematical statements can be manipulated to find conclusions.

Axioms	Addition	Multiplication
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverses	$a + (-a) = 0$	$a \cdot a^{-1} = 1$
Commutativity	$a + b = b + a$	$a \cdot b = b \cdot a$
Associativity	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributivity	$a \cdot (b + c) = a \cdot b + a \cdot c$	$(a + b) \cdot c = a \cdot c + b \cdot c$

Figure 2-9 Field Axioms in Algebra

A graph is a useful formal system to model complex systems with many interacting parts. A graph defines a set of vertices V and edges E between vertices, following $S_{Graph} = (V, E)$.[14] For example, Figure 2-10 shows the graph with $V = \{a, b, c, d, e\}$ and the edges are defined as ordered pairs. Edges can represent an arbitrary relation and can be directional or non-directional depending if the pairs go both ways, like *(a, b)* and *(b, a)*. Graphs can also be further generalized into hypergraphs, which allow edges that connect to any number of nodes. There are many other properties of graphs, such as being connected or the degree of centrality, which relates to the average number of steps required to reach all other nodes. Graphs can represent various mathematical structures and be useful in designing more resilient networks.

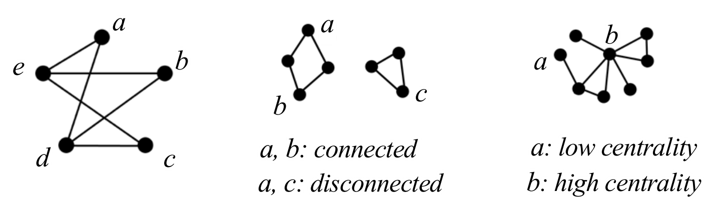


Figure 2-10 Graph Systems

A model, defined $S_{Model} = (U, \sigma, I)$, describes statements in a formal language, like logic or math, that are interpreted as true. A model considers a universe of possible values, called the domain of discourse *U,* along with the signature *σ*, which is a set of symbols that represents constants, functions, and relations. For example, a model of math could have the domain of the real numbers, *U = {Real numbers}*, and a signature of terms $\sigma = \{0, 1, +, -, \cdot, <, \ldots\}$. The interpretation relation *I* assesses if a statement *p* happens to be true or not for a set of axioms that applies to the elements *U* and terms *σ*. *I* is a model of *p* when the proposition *p* is true in *I.* Models can be used to assess meta-level properties, such as whether a theory is consistent and free of contradictions, and provide insight for interpreting truth in a system.

A summary of various kinds of formal systems, from logic to math, is shown in Figure 2-11. These systems are all expressions of a general system *S = (E, R)* with different elements *E* and relations *R*. These formal systems also have many types of subclasses specified by certain relations (e.g. propositional logic versus first order logic). Additionally, many of these formal systems have similar underlying patterns and can have mappings to interchange from one to another.

System	Equation	*E* Elements	*R* Relations
General	$S = (E, R)$	$E = \{E_1, E_2, E_3, \ldots\}$	$R = \{R_1, R_2, \ldots\}, R_r \subseteq E^n$
Formal Language	$S = (A, X, R)$	*A = alphabet {a, b, c, …}* *X = string(s) or sentences*	*R = {language relations, grammar, inference}*
Automata (Finite)	$S = (A, Q, Q_i, Q_f, R)$	*A = alphabet* *Q = set of states* Q_i = *initial;* Q_f = *final*	$R = Q \times A \rightarrow Q$ *{relation to new state}*
Logic	$S = (P, O, I, Z)$	*P = propositions {p, q, …} which are true or false*	*O= {operators: ∧, ∨, …}* *I = {inference rules}* *Z = {axiom rules}*
Math	$S = (E, O, Z)$	*E = {integers, real numbers, fields, …}*	*O= {operators: +, -, … }* *Z = {axiom rules}*
Graph	$S = (V, E)$	V = *vertices* $\{v_1, v_2, \ldots\}$	*E= {edges connect two given vertices}*
Models	$S = (U, \sigma, I)$	*U = Universe, Domain* *σ = Symbols in theory*	*I = {Interpretation function of truth}*

Figure 2-11 Overview of Formal Systems

Universal Potential

The universal set U, or domain, establishes the broadest set of a system's elements. For example, with the two sets $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$, a universal set could be $U = \{1, 2, 3, 4, 5, 6\}$. Taking a broader perspective, the universal set could be extended to the natural numbers $U = \{1, 2, 3, \ldots \infty\}$, or even the continuous real number line. From a graphical perspective, the universal set U is the outer box which contains any given sets, like A, and their compliments, like A^C, following $A^C = U - A$. When considering a universal set of logical outcomes, the set A can represent a proposition p and A^C is the negation $\neg p$, following $\neg p = U - p$.

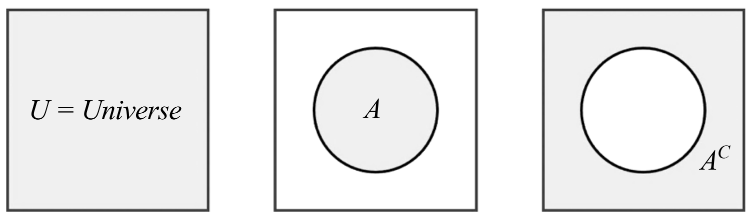


Figure 2-12 Universal Set

The universal set can be thought of as the background needed to pose a system, like the paper needed to draw a given model. Prior to distinguishing particular objects, the universal set is like an undifferentiated potential. With no specific elements defined, a system can be posed with empty elements $E = \{\}$ and empty relations $R = \{\}$, establishing the universal set as empty $U = \{\}$. In this case, the universal relation (relation with all elements) will equal the empty relation (relation with no elements). Once elements are included, the universal set differs from the empty set and distinguishes certain parts. Even with a closed universal set, there is an extreme openness for any scenarios to be created within.

The notion of a universe of discourse is essential in using logical quantifiers, like *for all* $\forall$ and *there exist* $\exists$. For all $\forall x$ means that for all x that are members $\in$ of the universal set U, the property, or predicate $P(x)$ is followed, written $\forall x \in U, P(x)$. There exist $\exists x$ means that at least for one of x that is a member of U, the predicate $P(x)$ holds true. The universal set provides the boundary of all possible outcomes. Logical systems that use quantifiers, including arithmetic rules extending to all numbers, are called first-order systems. Extending further, second order logic applies quantifiers on the predicates themselves and the relations between predicates.

Another approach to define a universal set is through a behavioral view. The "universum" U set is defined as the totality of states a system could be in prior to applying laws, and the behavior B, is the subset of outcomes of a given model, following $B \subseteq U$.[15] For example, when modeling the Earth's rotation around the Sun, the universum could include a continuous set three-spatial variables and one time variable $U = (x, y, z, t)$ and the behavior of the Earth's orbit would be the subset of values a model predicts. The behavioral approach to model system is powerful as it expresses the underlying relations as a single set B. For a general system with elements E, a universum can be defined as all E permutations, $U = E^*$. When the relations can be expressed as following one overall behavior, the system follows the definition $S \subseteq E^*$. This is analogous to the definition of a formal language with alphabet A as $S_{Langauge} \subseteq A^*$.

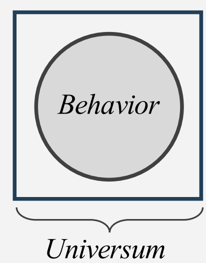
Another way to define the limits of behavior is by possible versus impossible physical transformations. Constructor theory distinguishes all possible transformations allowed by the laws of physics (done by "constructors") versus transformations that are not possible by the laws of physics. This differs from a Newtonian approach that focuses on finding one answer of the laws of motion, such as a specific orbit of Earth. From a constructor theory point of view, it is also possible that tools are constructed to alter Earth's trajectory, or accomplish any transformation allowed by physics. When taking a behavioral approach to constructor theory, the universum is the space of all possible physical transformations, and the behavior is the subset of transformations allowed by the laws of physics. Specific models and other tools can then assess which transformations are more likely to occur in certain scenarios.

The universal set may be posed to contain all sets, but this leads to contradictions. A set X containing all sets would contain its own power set (all subsets of X). However, this leads to contractions because a set should always have a lower number of elements than its power set.[16] Formal systems take a variety of approaches to fix these contradictions, such as introducing a "proper class", a collection of sets that cannot be elements of other classes. The class of all sets would not contain itself and correct these contradictions. The "categories" of category theory are also classes, so one can study the category of all sets. Similar to the set of all sets, contradictions arise in considering the system of all systems.[17] In a general system, it is often beneficial to consider a system S, elements E, and relations R, as different types of classes, or categories, to avoid self-referencing paradoxes.

Example 2.7
Behavioral System

Systems can be defined via the Universum U, the totality of events prior to applying laws, and the Behavior B, the subset of these outcomes that are in a given model.

$S_{Behavioral} = (U, B)$
U = *Universum of possible states*
B = *Behavior of model*, $(B \subseteq U)$



Relational Elements

> **Example 2.8**
> Interdependent Words
>
> A word only gains meaning through referencing other words, which also reference other words, with no objective truth.
>
> *Simple* = *Easy to do or understand*
>
> ↳ *Easy* = *Without effort or difficulties*
>
> ↳ *Without* = *In the absence of or outside*
>
> ↳ *In* = …

Defining an element in a formal system is accomplished by relating to other elements. A single term like *apple* requires a system of words to be defined, such as "I like *apple* pie". In turn, each of these words are defined by referencing other words. This can also be seen in the definitions of physics concepts, like *energy*, which hinges upon its relations to other quantities, like *distance*, *time*, and *mass*, which hinge on other terms. Number theory even defines numbers as the extension of a proposition, meaning that the term *four* only makes sense in statements such as, "There are *four* drawers."[18] While it may seem that elements like *four*, *energy,* or *apple* exist in isolation, relations to other elements are required to produce meaning.

Identity itself even arises in a relation-centric process. Identity is defined as a binary relation that maps an element to itself $x \longrightarrow x$, such as $\{(1,1), (2,2), (3,3), \ldots\}$. In the identity mapping, the pair $(1, 1)$ depends on connecting only to itself ($1 = 1$) and not connecting to all others in a domain ($1 \neq 2$, $1 \neq 3$, ...). Even defining one element $\{a\}$ requires the identity relation, creating the system $S = (\{a\}, \{(a, a)\})$. Essentially, in order to say anything about an element, even that it is equal to itself, requires a system of relations. Departing from a parts-based view that focuses on defining isolated elements, systems science studies the network of relations between elements.

Math and logic essentially studies the relations between things, not what things actually are. As formal systems are valid by virtue of form rather than content, they do not provide knowledge of what *"x"* is or should be, but rather how *"x"* relates to other variables, like $x = \frac{1}{2}y$. For example, geometric ratios are defined by the relations between multiple distances and an object *"x"* is defined as length *1* by being half as long as another object *"y"* of length *2*. Instead of posing isolated parts that exist in independence, a system-based view poses a system, or relations between elements, to model a given part.

Contextual and relation-based definitions come about in modeling nature. In quantum physics, individual particles can only be measured and observed via interactions with other particles. Quantum particles can also be entangled, which means that measuring the state of one particle can instantly effect other particles.[19] This means that the smallest pieces of matter in the universe cannot be identified in isolation and must be understood as a network of relations. More broadly, the complex physical, biological, and informational patterns in the world must be modeled through interdependent relations, where the identity and meaning of a given part arises from interaction.

Relations are central in category theory, which studies how mathematical objects map to one another. A category consist of mathematical objects (sets, geometric spaces, algebraic fields, etc.) and the relation of morphisms (mappings written as →) that follow identity and composition. Categories can include other categories as objects and the morphisms between categories are called functors, which also follow identity and composition rules. Morphisms can be further extrapolated into natural transformations, which are structure preserving mappings between two functors, as shown in the example in Figure 2-13 (identities and labels omitted). Category theory provides a very general way to study underlying relations within and between mathematical structures.

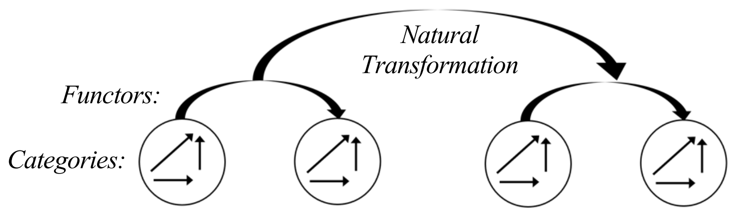
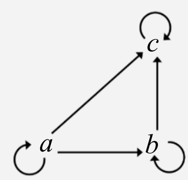


Figure 2-13 Categories and Morphisms

> **Example 2.9**
> Category Theory
>
> A category is a system with the elements of mathematical objects *(e.g. sets, fields, other categories, ...)* and maping relations, called morphisms *(→)*, that include identity *(a → a)* and composition *(if a → b & b → c, then a → c)*, like the graph below. Category theory can embed mathematical structures into graphs and map from one model to another.
>
> 
>
>
> $S_{Category} = (E, R)$
> *E = Objects*
> *R = Morphisms, must include identity and composition*

Relation-centric definitions arises in the Yoneda lemma, a fundamental result in category theory. The Yoneda lemma states that an object's map can be fully defined by how other arbitrary objects relate to it. This can be illustrated by a sphere. Objects on a plane, like a line, circle, and triangle can be mapped to the sphere, but will be curved to preserve their structure, as shown in Figure 2-14. Following the Yoneda lemma, the sphere's map can be fully acquired by knowing how arbitrary objects map to it, even without knowledge of the sphere itself. More formally, the Yoneda lemma can be written as $F_A = Nat(h_A, F)$, where F_A is a functor to the target object and is equal to $Nat(h_A, F)$, the structure preserving natural transformation from arbitrary objects to the target object.[20] The result, that a map to an object is fully definable just by the relations to other objects, has deep philosophical implications about identity.

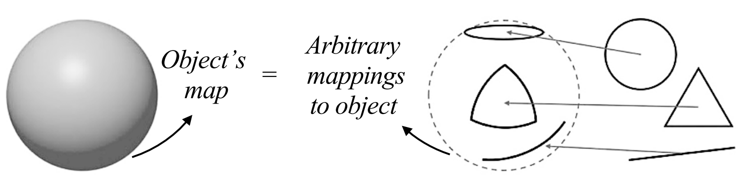


Figure 2-14 Yoneda Lemma

Models and Metalogic

A model, which is a set of statements in which a given formal system is interpreted as correct, can be studied in its own right and has distinct properties. In studying models, it is helpful to clarify the difference between the object language and the metalanguage. For example, if *A* and *B* are sentences of an object language $S_{Language}$, a metalanguage references the term $S_{Language}$ to study properties about the object language itself. The object language studies how to use a given formal system's symbols and operators, while the metalanguage is used to study the structure of the formal system. One could work within the rules of a mathematical system, like arithmetic for example, without ever considering the metatheory properties of the model.

> **Example 2.10**
> Embedded Metalanguage
>
> Object languages can also embedd an metalanguage. The English language has the words *noun, verb* and *word*, which describe concepts about the English Language. Gödel numbers, which assign mathematical statements to numbers, are another example of an embedded metalanguage.

$S_{Language}$ = {*Sentences A, B, C, …*} $S_{Metalanguage}$ = *Properties*($S_{Language}$)

Object language *(Within Model)*   Metalanguage *(About Model)*

Figure 2-15 Language vs. Metalanguage

Metalanguages provide clarification for defining truth. Alfred Tarski's semantic theory of truth, introduced in 1933, states that the object language "*p*" is true if in a metalanguage *p* is true.[21] For example, "*rain is wet*" is true in an object language if *rain is wet* is posed to be true in a metalanguage. Essentially, a models internal truth requires that it is externally assumed to be true. Tarski's undefinability theorem states that truth cannot be defined within a formal system (at least first-order logic) and requires a stronger metalanguage.[22] For example, arithmetic truth cannot be defined with arithmetic.

Mathematics can illustrate the difference between an object language and metatheory. The object language of math studies the results generated from manipulating rules, like in $2 = 1 + 1$. On the other hand, metamathematics studies what can be known about the model itself, such as if a model is *sound* and does not lead to false conclusions, like $x = 1$ and $x \neq 1$. Properties of models include being *sound*, *consistent*, and *complete*, which are defined in Figure 2-16.

Sound	*A proof system cannot lead to false conclusions*
Consistent	*A proof system proves no contradiction*
Complete	*A proof system can lead to any true conclusion*

Figure 2-16 Properties of Models

Not all formal systems are fully sound, consistent, and complete—even if they may appear so at first. For example, there are contradictions in the basic axioms of set theory, called "naïve set theory". Cantor's paradox revealed that the set of all sets is not well-defined because it would also contain itself, leading to contradictions. Also, Russell's paradox shows that the set of all sets that are not members of themselves leads to contractions. Systems like Zermelo-Fraenkel set theory and Type Theory introduce new axioms to correct these contradictions and create a consistent form of set theory.

Even if a proof system is free of false conclusions (sound) and free of contradictions (consistent), this does not mean that the system can prove any true statement (complete). While, zero-order logic, called propositional logic, is complete and all statements always have provable yes or no answers, first-order logic, which has quantifiers for variables like *for all x*, is incomplete. This means there are true statements that are impossible to solve.

In the early 1900s, logicians were working to classify mathematics into a complete system where each proposition would have a provable true or false result. To the dismay of these logicians, Gödel's 1931 incompleteness theorem, proved that no math system powerful enough for arithmetic (or first-order logic) can prove itself complete.[23] This discovery re-affirmed that math contains true statements that are impossible to solve, even though math is sound and consistent. For example, the continuum hypothesis, which is a problem in set theory that asks if there is a set size between the size of the integers and continuous real number line, has been proven to be undecidable from initial axioms.[24]

A commonly explored metatheory that connects to natural sciences is metaphysics, which works to describe first principles that cannot be tested in physics itself, including concepts such as substance, cause, time, and space. While physics can explain how measurable quantities like mass, time, and speed relate to one another, physics does not explain why fundamental concepts exist in the first place. Metaphysics, on the other hand, explores questions that are assumed prior to the possibility of empirical testing.

Throughout history, many different metaphysical models of reality have been posed. Philosophers like Plato and Hegel posed extravagant metaphysical systems. However, there are usually inconsistencies or inefficiencies in applying one given metaphysical model to describe all real-world situations. While these models can have degrees of insightfulness, no metaphysical system has ever been agreed upon, nor can ever be proven as, the true model of reality.

In the 19th century, philosophers largely transitioned away from metaphysical theories to developing analytic philosophy, which studies how questions can be proven in a given logical model. This revolution, called the linguistic turn, reoriented the focus of philosophy to discuss the process associated with logical truth for a given formal system and denied the ability to know if reality truly follows a model. The linguistic turn has shaped modern philosophy to study the process by which any formal model can be analyzed.

Systems theory takes a similar approach to analytical philosophy, as it does not search for one ultimate model of nature, but instead clarifies the meaning of truth in any possible theoretical system that can be posed. Systems theory does not have an external definition of truth, but rather each system has its own truth, when assuming the axioms are true in a metalanguage. Science then uses evidence-based methods to correlate formal theories with real-world phenomena within certain degrees of accuracy.

While analytical philosophy has dropped the hunt for the single ultimate model of reality, scientists often focus on predicting nature from the bottom-up. For example, biologists often work to explain life solely through the underlying chemistry. While the unity of science takes the view that all the rules of nature are compatible with one another (for example, that biology does not violate physical behaviors of matter), complex systems can have irreducible higher-level patterns that are impossible to efficiently predict, showing the limitation of a purely bottom-up approach. Higher-level theories, like chemistry, biology, and so forth, are often required to create useful models of nature, rather than just using physics for everything.

Systems theory has different goals compared to a parts-based metaphysics. Instead of seeking a singular and fixed model that perfectly predicts all scenarios in the universe, systems theory works to provide simple expressions to describe any logical model. Systems theory does not pose one fixed, unequivocally correct, model of nature, even if the scientific models of nature strive to be fully consistent. Systems theory instead takes a metalogic approach to consider how any kind of model can be used. Furthermore, while the success of a parts-based and reductionist approach is determined by completeness and predictability, a successful systems theory must include the possibility for models that are incomplete and irreducible. Other differences between the parts-based and systems approach are summarized in Figure 2-17. Systems theory refocuses the search for a single fixed model of nature toward how models themselves work—simple to complex.

Type:	Parts-Based Metaphysics →	Systems-Based Metalogic
Goal:	*Find one model that can explain all the patterns in the universe*	*Find what applies to all systems used to model any given pattern*
Explanation Power:	*Explain all scenarios in the universe in a single reducible model*	*Explain all systems of representation that can be reducible or irreducible*
General Methods:	*Model is complete, consistent, and all questions are decidable*	*Models account for simple to complex systems, which can be undecidable*
Measure of Failures:	*Fails if it produces at least one inconsistency or paradox*	*Fails if it is does include any given model that can be proposed*
Form of Success:	*Find the simplest list of entities and relations for the entire universe*	*Identify generally applied and unifying principles of any given system*

Figure 2-17 Reductionist vs. Systems Approach

Summary

This chapter provided foundational definitions for a general system. Set theory can be used to define an arbitrary system as a collection of elements and relations between those elements. These relations can represent logical operators, like if-then statements, as well as mathematical operations, like addition and subtraction. Any formal system, from math, logic, graphs, and automata, can be created by specifying elements and relations. Within a system, elements do not possess isolated identities, but are rather defined by the interdependent relations to other elements. As a meta-discipline, systems theory provides insight into what can be known about formal models, such as the possibility for incompleteness. In contrast to the traditional parts-based and fully reducible approach, systems theory presents an open-ended way to approach nature's patterns and knowledge that includes complexity, incompleteness, and irreducibility.

Chapter 3 Emergence

Example 3.1
Flock of Birds

Flocking patterns can be studied with emergent, coarse-grained models that only considers the behaviors of large collections.

Emergent models provide a useful way to describe arising properties in systems. More formally, a higher-level emergent model is mapped from limiting the domain (or universal set) of behaviors covered by a lower-level theory. One way to limit a domain is by coarse-graining, which only considers collective properties. Temperature is an example of a coarse-grained emergent model as it only considers the average energy without knowing each particle's kinetic energy. Temperature can then be related to other emergent measures like pressure, and entropy. Emergent models propose numerous measures to analyze the world, relevant for particular domains of science.

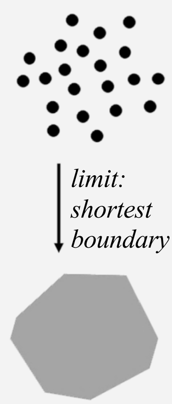
$$limit(S_{Lower}) \rightarrow S_{Higher}$$

Figure 3-1 Equation of Emergent Theories

Emergence plays a critical role in science and connecting lower-level models, like physics, to higher-level models, like biology. A critical insight is that emergent models provide consistent and parallel descriptions of lower-level models, so both are valid and meaningful. This means that higher-level models should never introduce something fundamentally new, even if different or surprising. However, the lower-level rules of complex systems cannot always be efficiently mapped to higher-level results. So, while the domain of biology may be a subset of physics, this does not mean physics can efficiently predict biological patterns.

Example 3.2
Shortest Boundary

Nodes create an emergent shortest boundary length. The boundary limit contains all the nodes, but does not specify each node.



Emergent Models

The center of mass is an example of an emergent model. The lower-level method to model the gravitational force at a point from a set of particles is to consider each mass and distance, (m_1, r_1), (m_2, r_2), … . However, a more simple way to do this calculation is to sum Σ all the particles to create a total mass M_{cm}, and to find the distance from the center of mass R_{cm}, following the formula in Figure 3-2. From here, the total gravitational force outside the boundary only requires (M_{cm}, R_{cm}) rather than all the particles. In this example, both the lower- and higher-level models have the same type of measures of mass and distance. Additionally, the center of mass limits the details of internal regions so is not valid in all cases, but is still very useful.

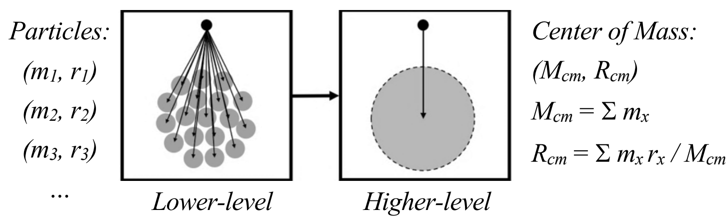


Figure 3-2 Center of Mass Model

Thermodynamics is another emergent model arising from the collective behavior of many particles of mass (m), position (r), and velocity (v). Thermodynamics introduces new measures like *pressure (P), temperature (T),* and *density (D)* that relate to one another in equations such as $P \propto T \cdot D$ for an ideal gas. These measures, and others like entropy, provide an efficient way to model system-wide states of large collections without information about each particle. Studying thermodynamic systems, which was called "the working substance" by early pioneer Sadi Carnot in the 1800s in relating entropy to the useful output of work, was a critical step in developing the notion of a "system" in science.[25]

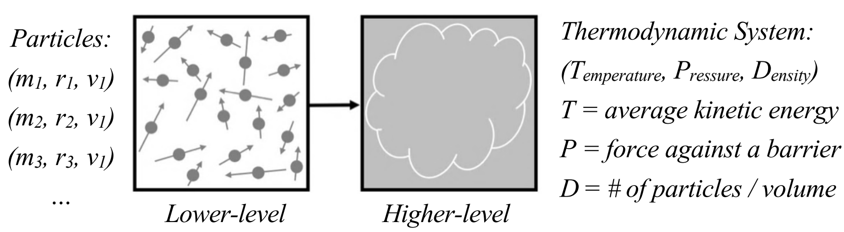


Figure 3-3 Thermodynamic System

Emergence arises in the different phases of matter, such as solids, liquids, and gases. The higher-level measure of phases considers the structural patterns and behaviors of many lower-level particles, as shown in Figure 3-4. Matter in a solid state, like ice, resists forces and maintains a volume. Matter in a liquid state, like water, is not rigid, but still maintains a given volume. Matter in the gas phase, like steam, will spread apart and not maintain a given volume. The type of phase can lead to vastly different macro-level behaviors and phases can change with sharp transitions at specific temperatures. Analogs to phase changes occur in other complex systems, such as models of animal flocking and swarming patterns.[26]

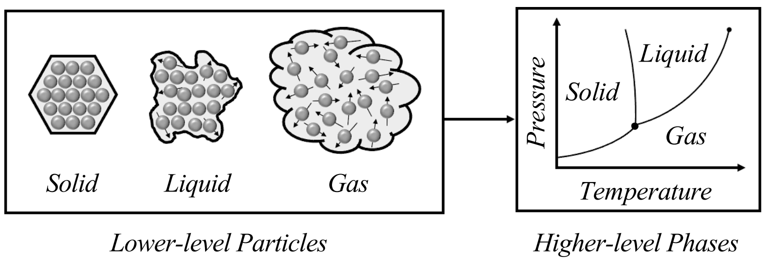


Figure 3-4 Phase Transitions

Economic models provide another interesting case of emergence. At the lower-level, economics considers individual monetary transactions for goods and services. Many individual actions can then be summed up to create system-wide patterns, such as total supply and total demand. A variety of collective patterns emerge in economic systems as independent agents attempt to optimize transactions, such as equilibrium points, where supply and demand equalize around a given price. Economic systems can also exist in out-of-equillbrium states and exhibit complex behavior that is not deterministic, predictable, and mechanistic, but rather process dependent, adaptive, and always evolving.[27]

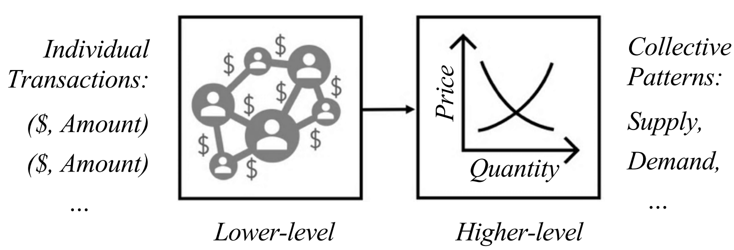


Figure 3-5 Economic Systems

Levels of Theories

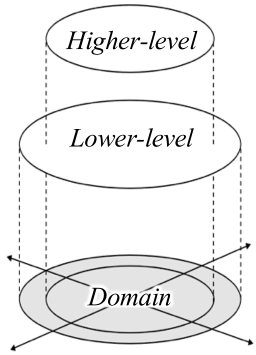


Figure 3-6 Domains of Emergent Models

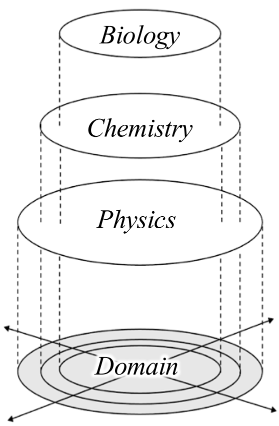


Figure 3-7 Domains of Natural Sciences

Higher-level theories have a smaller domain, or set of possible scenarios, compared to lower-level theories.[28] For example, the property of temperature is not valid with a small number of particles, while the lower-level particle-based model is valid. Another example is that the lower-level model of relativity includes scenarios near the speed of light, while the emergent model of Newtonian mechanics does not.[29] In general, the domain of a higher-level model is a subset, or part of, the lower-level domain, as shown in Figure 3-6.

Natural sciences are largely categorized as nested emergent levels. Physics strives to model energy, matter, and spacetime at the lowest fundamental level. Chemistry provides higher-level models of the world, limited to matter above the molecular scale. Biology is limited to the subset of chemical systems with living properties. Further higher-level models can be defined, like psychology and sociology. While higher-level theories, like biology, may propose surprising rules that are not efficiently predictable by lower-level theories, like physics, these models should, in principle, be compatible and consistent.

Lower-level and higher-level models of the same system should always agree on the underlying behavior when both can be applied. For example, thermodynamic models should produce the same results as modeling individual particles. This equivalence can be shown with a mapping process →, that transforms inputs to outputs. Mapping how particles change over time, then deriving the state of the gas ($L_0 \rightarrow L_t \rightarrow H_t$) should be equivalent to first mapping the particles to gas, then modeling how the gas changes over time ($L_0 \rightarrow H_0 \rightarrow H_t$), as shown in Figure 3-8. Both levels are parallel descriptions of the same behavior.

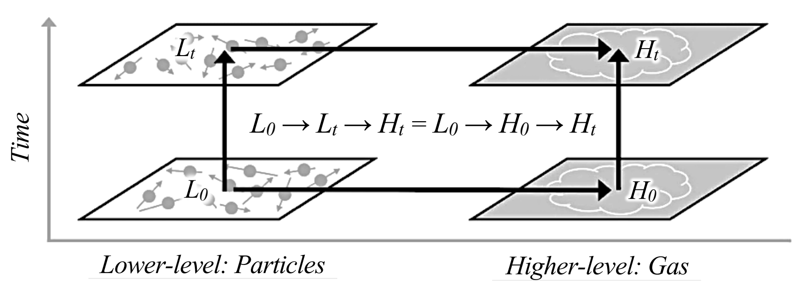


Figure 3-8 Equivalent Mappings

While some higher-level models can be exactly derived, many emergent mappings do not have efficient algorithms. For example, the properties of gases can be exactly derived from many random particles, but the computation halting problem has irreducible higher-level properties that cannot be predicted with efficient algorithms. Computers can support modeling emergent properties, but often require unfeasible resources to find solutions without quick algorithms. Also, the mappings between theories may only be partially solvable (e.g. only some of chemistry is predictable by physics) or have no known solutions (e.g. how cognitive experiences and consciousness arises), even if they are expected to describe the same behavior.

Higher-level theories can be, and are often, discovered without knowledge of a lower-level model. For example, temperature and pressure were understood before the discovery of the lower-level theory of particles. Over history, many higher-level theories have been subsequently mapped to broader lower-level theories, like linking inheritance trends to DNA structures.[30] However, many higher-level models exist independently and may never be derivable from lower-levels.

Emergent models can have a variety of connections between different levels of theories. The trivial case is that one lower-level theory leads to one higher-level theory. One lower-level theory can also lead to multiple higher-level theories. For example, limiting the lower-level electromagnetic theory can lead to two separate theories of electricity and magnetism. Multiple lower-level theories can lead to a single higher-level theory. For example, both quantum mechanics and relativity simplify into classical mechanics, but these two theories are distinct and not consistent with each other.[31] Science works to create consistent and comprehensive theories of nature, but there are still large gaps and complexity hurdles for connecting emergent levels.

Example 3.3
Difficult Mappings

Not all mappings from lower-level models to higher-level models are efficient or fully solved.

$$limit(S_L) \xrightarrow[\text{Efficient}]{} S_H$$

(e.g. center of mass)

$$limit(S_L) \xrightarrow[\text{Nonefficient}]{} S_H$$

(e.g. halting problem)

$$limit(S_L) \xrightarrow[\text{Some Solved}]{} S_H$$

(e.g. predicting protein folding structures)

$$limit(S_L) \xrightarrow[\text{Unknown}]{} S_H$$

(e.g. cognition arising in biological systems)

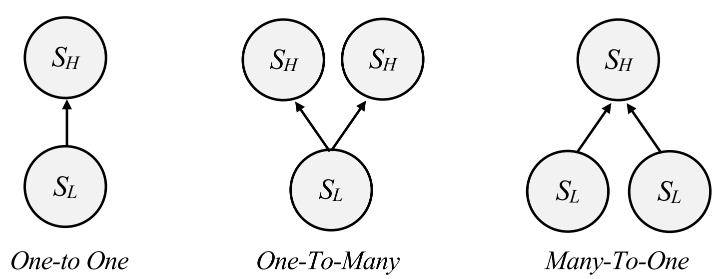


Figure 3-9 Emergent Theory Connections

Limiting Domains

Example 3.4
Simplifying Equations

Emergent theories can provide ways to model with fewer degrees of freedom. Chemical reactions formulas, like $2H_2+O_2 \rightarrow 2(H_2O)$ do not include subatomic states, (e.g. each proton consist of quarks) while still effectively predicting molecular processes.

Lower-Level Theory
{*Many quantum states*}
↓
Higher-Level Theory
{*Fewer chemical states*}

Emergent limits can be considered from a behavioral approach, which defines a system as $S = (U, B)$. The domain is a universum U of possible outcomes and the behavior B is the subset of outcomes predicted by a model, following $B \subseteq U$. Emergent models can describe limits of this unified behavior. The behavior described by a higher-level model is a subset of the behavior described by the lower-level within the universal behavior, following $B_{Higher} \subset B_{Lower} \subseteq B$. Subsets of behavior can also be assigned respective domains, $B_{Higher} \subseteq U_{Higher}$ and $B_{Lower} \subseteq U_{Lower}$, which are subsets of one another, following $U_{Higher} \subset U_{Lower} \subseteq U$.

An universum of values can be limited in many ways to study a subset of behavior, revolving around lowering the degrees of freedoms. One limit is to join "$\cup$" sets A and B and only consider their intersections $A \cup B$, which reduces the sets under study to one. The variables x and y can be limited by extending the x variable to infinity to establish $y_{x\rightarrow\infty}$. A universum of x, y, and z values can also be limited to x and y with a fixed z value, like $z = 0$. A set can be limited to cover a smaller range of values, such as $\{1, 2, 3\} \rightarrow \{1, 2\}$, which can be thought of as limiting the range of velocities or energies considered. While not all limits have a smaller output, the limits in emergence reduce the size of the output, called the codomain, compared to the input, or domain. Additionally, the limit's arrow goes left to right to an output codomain, which is called a colimit in category theory. In emergent limits, detail is lost in the colimit and there are no maps backwards to the initial domain.

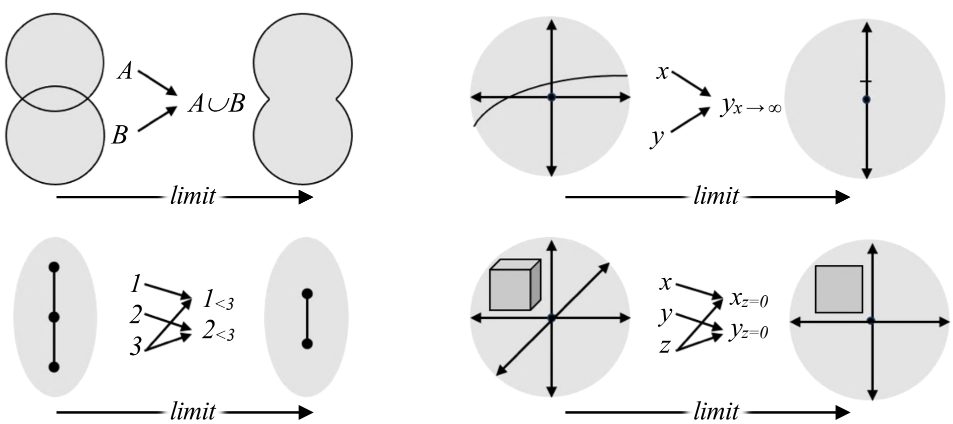


Figure 3-10 Limiting Domains

Emergence can be more precisely defined by introducing an interpretation map I, which transforms a system's behaviors into interpreted results following $I: S \rightarrow S_I$. The interpretation of a system, like $I(S)$, is a model and another type of system $S_{Model} = (U, \sigma, I)$ that interprets the underlying domain, like $S_{Behavioral} = (U, B)$. Emergence can then be defined as an interpretation that can only be mapped one way for a colimit following, $limit(I(S)) \rightarrow I(limit(S))$.[32] To understand this equation, it is useful to define a lower-level model as interpreting the system, $S_{Lower} = I(S)$, and a higher-level model as interpreting the system with a limit imposed, $S_{Higher} = I(limit(S))$. The emergence relation of $limit(I(S)) \rightarrow I(limit(S))$ can then be written as $limit(S_{Lower}) \rightarrow S_{Higher}$. An essential aspect of this definition of emergence is that the higher-level and lower-level models are different interpretations of the same system S. The behavior of the underlying system is always the same $S = (U, B)$, but certain interpretations under limits give the effect of emergent models.

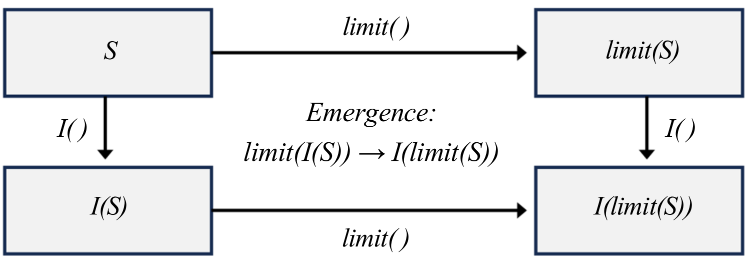
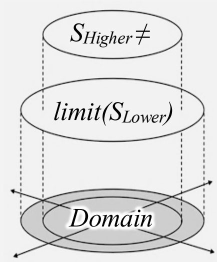


Figure 3-11 Interpretations and Emergent Limits

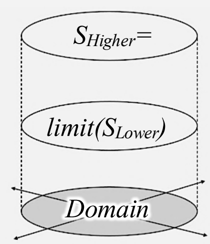
Emergence occurs when a limit is not preserved for a given interpretation and can only be mapped in one direction, following $limit(S_{Lower}) \rightarrow S_{Higher}$. Emergence models cannot be mapped in reverse, which entails the limit is not isomorphic and is unequal $limit(S_{Lower}) \neq S_{Higher}$. For example, the limit of the set $\{x, y, z\}$ can be mapped to $\{x, y\}$ as another model of the underlying behavior, but is unequal as $\{x, y, z\} \neq \{x, y\}$. In emergent models, the interpretation mappings must have a lower amount of detail than the full behavior. If the higher-level system loses no detail and can fully reconstruct the lower-level model, then the models are not emergent, but rather equivalent to an isomorphism, following $limit(S_{Lower}) \leftrightarrow S_{Higher}$ and $limit(S_{Lower}) = S_{Higher}$. One way equivalent models can occur is if the interpretation does not lose any detail about the system's behavior, which entails both sides will be equal to each other in the equation $limit(I(S)) = I(limit(S))$.

Example 3.5
Emergent Domains

Emergent models lose detail, are unequal, and reduce the domain over a given limit. Equivalent models lose no detail, are equal, and cover the same domain.



Emergent



Equivalent

Emergent Elements

Emergent theories often propose new measures, such as temperature, solidity, and homeostasis, to describe a system in completely different terms than lower-level theories. While a parts-based view may understand higher-level properties as just side-effects of a lower-level, a systems-based view recognizes emergent patterns as valid models in their own right. New measures that efficiently model natural systems without lower-level information are extremely useful. Additionally, giving levels more or less importance does not make sense if they are parallel descriptions of one reality, subject to different limits.

Emergence models can be classified by different types of elements. Homeostructural ("same" structure) emergence occurs when the elements in lower-level and higher-level models have the same type of measure, written $type(E_{Lower}) = type(E_{Lower})$. The center of mass is a homeostructural theory because the same elements of position and mass are used in both the lower-level and higher-level model. [33] Heterostructural ("different" structure) theories have different types of elements $type(E_{Lower}) \neq type(E_{Lower})$. For example, entropy and temperature use different measures than the lower-level theory of particles. Heterostructural emergent theories show how new descriptions that are wholly different can arise, like tensile strength and viscosity, when limiting to subsets of the domains.

$type(E_{Lower}) = type(E_{Higher})$	$type(E_{Lower}) \neq type(E_{Higher})$
Homeostructural	*Heterostructural*

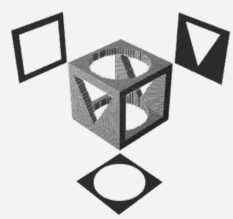
Figure 3-12 Emergent Element Types

While emergent measures can be different than lower-levels measures, they are not fundamentally new. Following the view that lower-level and higher-level models describe the same behavior means there should never be so-called "strong emergence", where emergent theories introduce something incompatible with lower-levels. Strong emergence is often referenced when asking if life or consciousness requires something else beyond the physical universe. However, following the notion that science identifies ubiquitous patterns of reality, rules of the whole universe, like physics, should always be compatible with rules of subsystems of the universe, like living and information systems. For example, computational circuits can present the emergent property of a software, but this does not require something else beyond the physics of the hardware.

The elements of emergent models can map to one another in a variety of ways. In coarse-grained theories, many lower-level elements map to a single higher-level element. Temperature and center of mass uses coarse-graining to define one macrostate. In fine-grained theories, each lower-level element maps to one higher-level element. For example, each particle of mass in the lower-level theory of special relativity maps to one particle of mass in classical mechanics. Another type is a one-to-many mapping, which is here called "hidden-order" as it hides details about the unifying structure. For example, there is only one lower-level quantum field for each particle type, but many separate higher-level particles. [34] Importantly, regardless of a given element's scope, there is always an underlying limit where the output domain's degrees of freedom is smaller than the input domain.

Example 3.6
Hidden-Order

Lower-level elements with hidden-order unifies many higher-level elements, such as the circle, square, and triangle all arising as shadows of a single 3-D object. Hidden-order emergence occurs in symmetry breaking, a physical phenomenon where lower-level states are symmetrical and unified, but break this symmetry to distinct higher-level states.



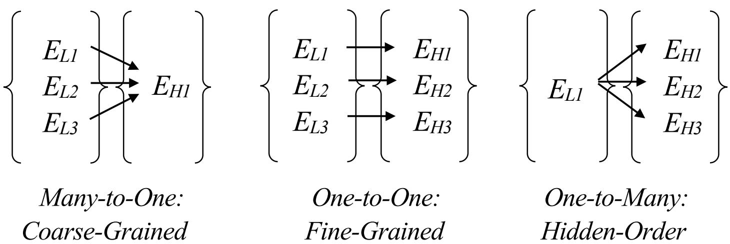


Figure 3-13 Emergent Element Mappings

A common type of emergence are multiple coarse-grained emergent theories that nest into one another. An example of this is in Figure 3-14, which shows three coarse-grained emergent theories. The lowest level S_1 is coarse-grained to S_2, which is coarse-grained to S_3, each with their own set of elements. Nesting elements in emerging theories happen frequently in the fields of natural sciences. For example, quarks compose atoms, atoms compose chemicals, chemicals compose organisms, and so forth.

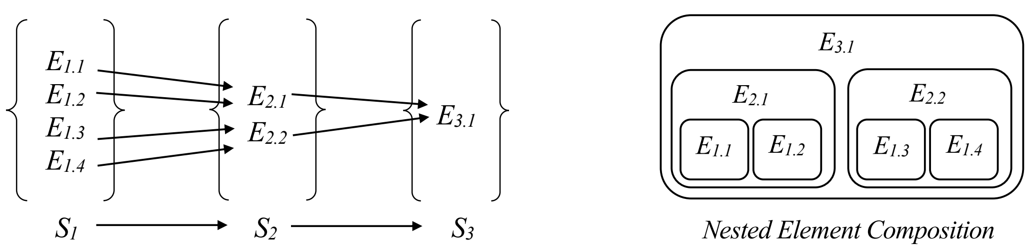


Figure 3-14 Nesting Emergent Elements

Generative Effects

A classic example of emergence is when a model of the whole is different than the sum of its parts. These generative emergent effects can be illustrated by a join "$\cup$" relation, which joins two systems, overlapping the sets of behavior. The joined system (*e.g.* $S_2 \cup S_1$) only contains the subsystems *(e.g.* S_1, S_2*)* and no other elements or relations. When defining a system as $S = ((S_1, S_2), \cup)$, which contains the elements of two subsystems (S_1, S_2) and the join operator; generative effects occur when $I(S_1) \cup I(S_2) \neq I(S_1 \cup S_2)$.[35] This formula can be found by applying a joining $\cup$ limit in the inequality of $limit(I(S)) \neq I(limit(S))$. While the model of the whole may not equal joining the model of the parts, the whole is always mapped from the parts, following the emergence equation $limit(I(S)) \rightarrow I(limit(S))$, or $I(S_1) \cup I(S_2) \rightarrow I(S_1 \cup S_2)$. Generative emergence is essentially the definition of nonlinearly but pertains to models themselves rather than particular functions. In equivalent and linear models, the joined parts is equal and isomorphic to the whole, following $I(S_1) \cup I(S_2) = I(S_1 \cup S_2)$.

Equivalent, Linear	Emergent, Nonlinear
$I(S_1) \cup I(S_2) = I(S_1 \cup S_2)$	$I(S_1) \cup I(S_2) \neq I(S_1 \cup S_2)$
$I(S_1) \cup I(S_2) \leftrightarrow I(S_1 \cup S_2)$	$I(S_1) \cup I(S_2) \rightarrow I(S_1 \cup S_2)$

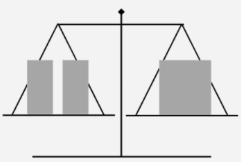
Figure 3-15 Equivalent and Emergent Models

Generative emergent effects can be seen in large language models used in artificial intelligence software. This kind of software uses a large amount of interconnected parameters to transform inputs, like text questions, into specific outputs, like text responses. The parameters of the artificial network can be thought of as smaller interpretational models that work together for the model's total effect. Interestingly, many of the effects of the large language models are not predictable from linearly scaling the smaller models and parameters.[36] These generative effects are similar to sudden phase transitions. After adding many parameters, the whole system reaches a tipping point, or phase transition, where it becomes much more accurate at solving complex problems. Even though the whole model is mapped from joining the parts, the total model output and ability to solve problems does not equal linearly joining the models of the parts.

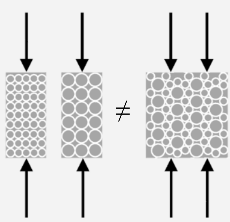
Example 3.7
Nonlinear Effects

Models, like summing the mass $M()$ of parts, are linear and add in superposition, while compressive strength $C()$ does not equal the sum of the parts and depends on the interrelated structure.

Equivalent, Linear
(Mass of parts adding)
$M(a) + M(b) = M(a+b)$



Emergent, Nonlinear
(Compressive strength)
$C(a) + C(b) \neq C(a+b)$



A critical feature that allows generative effects to come about is interconnected subsystems. This can be shown by taking an interpretation I where the details of the behavior's states are forgotten about and only the number of total states are counted. If the states of two systems are $S_1 = \{a, b\}$ and $S_2 = \{c, d\}$, the union equals $S_1 \cup S_2 = \{a, b, c, d\}$. This mapping is not emergent and adds together in a linear fashion, $I(S_1) \cup I(S_2) = I(S_1 \cup S_2)$, as the number of enumerated states can be put in a one-to-one correspondence of equal length, $4 = 4$. In contrast to the first case, when $S_1 = \{a, b\}$ and $S_2 = \{b, c\}$ the union shares the b state and equals $S_1 \cup S_2 = \{a, b, c\}$. Due to the fact that the interpretation mapping I removes the state's detail and only counts an arbitrary number of states, the model is emergent and the interpretation of the whole does not equal joining the interpretations of the parts, $I(S_1) \cup I(S_2) \neq I(S_1 \cup S_2)$, and $4 \neq 3$.

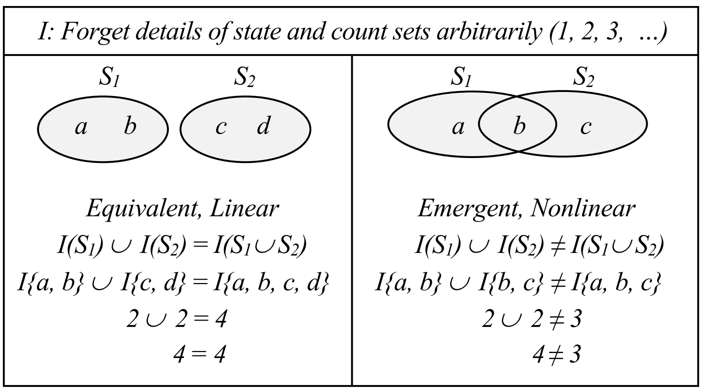
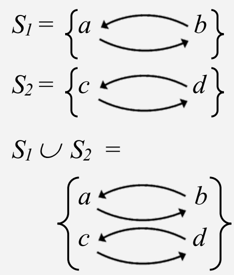


Figure 3-16 Emergent Effects in Joining Systems

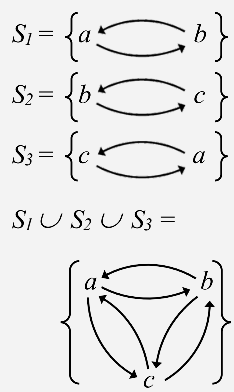
The necessity for two subsystems must be interlinked for generative emergence to arise is apparent in adding entropy. If the states of the subsystems are not related, the entropy of the whole system equals the sum of the entropy of the subsystems. However, if the states of the subsystems mutually depend on one another, then the entropy of the whole may not equal the sum of the parts. When subsystems share a set of behaviors, there exist the possibility to describe the joined behavior in a more efficient fashion. This creates the effect where joined model covers a smaller domain of behavior, and is unequal, to the model of the parts. Joined higher-level emergent models never covers a larger domain of behavior than lower-level model, and if the domains are equal in size, the models are equivalent and linear.

Example 3.8
Interacting Subsystems

Joining ∪ systems combines unique elements and relations. When subsystems are not related, models of the joined whole are linear and equivalent.



When subsystems are interelated, it is possible for the model of the whole to be different than joining the parts.



Irreducible Mappings

A critical dimension of emergence is that some limits are irreducible, meaning there is no finite algorithm to complete the mapping over a certain number of steps, leaving questions about the higher-level measures unanswerable even when the lower-level rules are known. For example, consider that a lower-level system S_{Lower} is a given set of initial rules for a computer that may or may not halt, the limit is to extend these rules to infinite number of steps over time $(t \rightarrow \infty)$, and the higher-level system S_{Higher} is if the computer program halts or not. While some programs may halt, other programs would be running infinitely long and in general, this emergent property would not be reducible to a finite number of steps. It is important to distinguish that this irreducibility is not about the parts being inconsistent with the wholes, but rather about the inability to reduce a decision problem. Higher-level wholes are always, in principle, constructed by and mapped from lower-level parts. However, not every mapping algorithm is reducible to an output or can be efficiently solved.

$$limit(S_{Lower}) \xrightarrow[\text{Finite Steps}]{} S_{Higher}$$

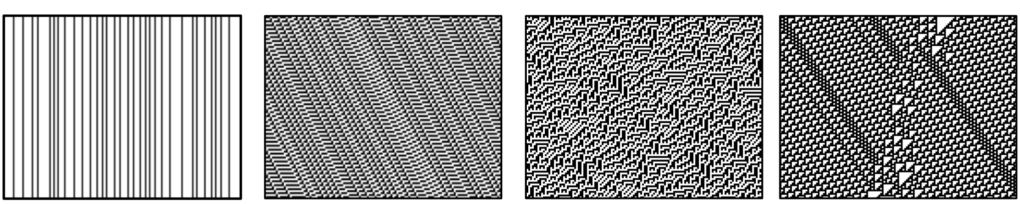
Reducible Emergence

$$limit(S_{Lower}) \xrightarrow[\text{Infinite Steps}]{} S_{Higher}$$

Irreducible Emergence

Figure 3-17 Reducibility and Irreducibility

Even short sets of lower-level rules can create irreducible higher-level patterns. For example, the elementary cellular automata program outputs rows of black and white cells, the color of each new cell determined by the colors of the three neighboring cells in the higher row.[37] These patterns fall into different classes, some of which are highly complex. Class 1 rapidly converges to a uniform state and Class 2 converges to a repeating state, making them easy to predict. However, Class 3 is chaotic and Class 4 forms complex structures.[38]



Class 1: Uniform *Class 2: Repeating* *Class 3: Chaotic* *Class 4: Complex*

Figure 3-18 Cellular Automata Classes

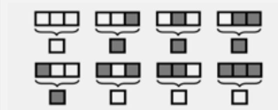
Many properties of cellular automata are irreducible, such as if the program will halt or how local patterns will evolve. These irreducible properties have no efficient algorithms to achieve precise results. Simulations need to be performed, potentially indefinitely, to test higher-level patterns generated from even simple sets of rules. The evolution of the Class 4 pattern Rule 110 displays other emergent properties, including the presence of "gliders", localized patterns that move diagonally across the grid. Additionally, Rule 110 can simulate an arbitrary computational process, which is a powerful and open-ended emergent property.[39]

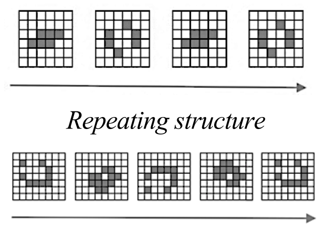
The irreducible patterns in automata can even contain self-organization and self-replication. The *Game of Life* version of cellular automata, introduced by John Conway in the 1970s, showed how complex, self-reproducing structures can be formed through simple rules based on the eight neighboring cells of a 2-D grid (up, down, left, right, and diagonals).[40] By repeating simple rules the automata can create repeating and organized structures, some of which are in Figure 3-19. John von Neumann built upon these ideas and showed how it is possible to create self-reproducing cellular automata structures that encode a strand of information, analogous to DNA, to reproduce patterns. These simple starting rules can generate emergent behaviors, like organization and self-replication, under the limit of many algorithmic procedures.

Some emergent patterns can be identified with brute-force solving, but this quickly becomes infeasible with even small collections. For example, consider assessing a collective property on a 10 by 10 black or white grid that needs to be tested individually with no quick algorithm. A brute-force method would require analyzing 2^{100} configurations, some of which are shown in Figure 3-20. This is a truly infeasible scale to calculate. Even if each configuration took one billionth of a second to compute, the calculation would take 100 trillion years. Many emergent states are near impossible to solve by brute-force. Even the snowflake is subject to this phenomenon—the orientations of a water molecules are simple to count in isolation, but a collection of water molecules in a snowflake has an immense size of possible orientations and a near zero probability of ever being duplicated in the span of the universe. Small collections of component rules can result in vast numbers of emergent possible outcomes, creating an irreducible myriad of unique outcomes.

Example 3.9 Rule 110

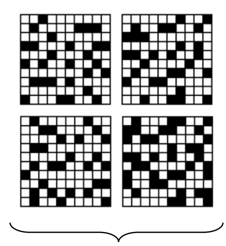
The Rule 110 cellular automata is a Class 4 pattern that can also simulate a computer.





1. *A live cell with 2 or 3 live neighbors lives.*
2. *A dead cell with 3 live neighbors becomes alive*
3. *All other live cells die and dead cells stay dead.*

Figure 3-20 Game of Life Cellular Automata



2^{100} Possible States

Figure 3-19 Brute-force Test

Multilevel Factors

A system's behavior can be simultaneously modeled by the lower-level effects to the higher-level collections, called bottom-up factors, and modeled by the higher-level effects to the lower-level parts, called top-down factors. For example, the lower-level rules of DNA produce the higher-level rules of proteins through bottom-up factors. At the same time, proteins can influence which portions of DNA is expressed by top-down factors. A bottom-up approach can model certain DNA mechanisms very well, but other long-range and long-term actions are better modeled via a top-down system of genetic interaction networks.[41]

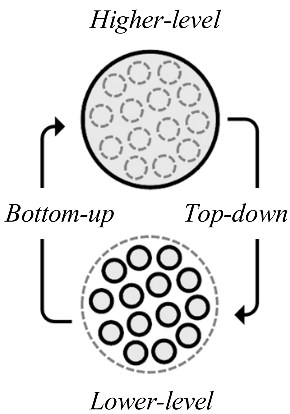


Figure 3-21 Bottom-Up and Top-Down Factors

Another example of self-hierarchy is the in the interaction of the mind and body. The body, brain, and neural networks create the upward, or bottom-up, factors constructing the basis for the emergent property of mental states. Different mental states can be roughly measured as brain wave frequencies and be altered through ingesting certain compounds into the body. At the same time, mental activity can influence the internal chemistry of the brain and body, such as the placebo effect. Therapies directed toward addressing multi-level links between mind, brain and body can be particularly effective in treating chronic diseases.[42]

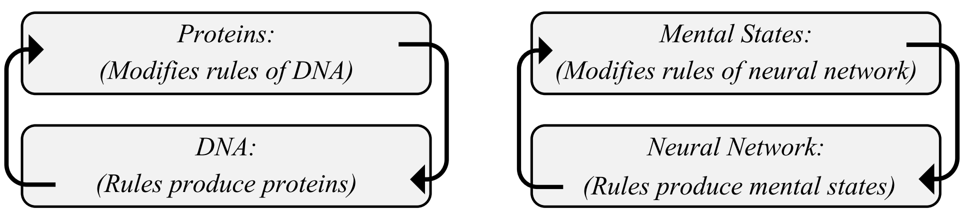


Figure 3-22 Self-Hierarchical Systems

A multilevel systems view of life considers both bottom-up chemical mechanisms and top-down environmental factors. This includes the influence of genes, or nature, as well as environmental influences, or nurture. Another top-down effect in biology is that animals in ecosystems do not always evolve to maximize individual benefits, but also evolve to improve symbiotic relations to other animals. This contrasts parts-centric notions in biology that genes are purely "selfish", and it shows that living systems have highly interdependent, top-down driven factors. A systems view considers models of upwards and downwards factors as parallel and alternative ways to understand the unified behavior.

Multilevel patterns can lead to seeming paradoxical feedback loops, where cause and effect are cyclically intertwined. A common example is the dilemma, “Which came first, the chicken or the egg?” This question asks how living systems arises if life is needed to create life in the first place. Cyclical causation is called a “strange loop” in the book *Gödel, Escher, Bach*. Geometric examples of strange loops are shown in Figure 3-23, including a Möbius strip where following a single surface will cover both sides. Another example is Penrose stairs, an impossible object where walking up the stairs will bring the walker back to the same place.

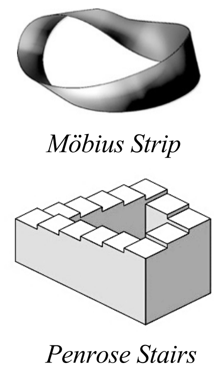


Figure 3-23
Strange Loops

The interaction between lower-level and higher-level systems plays a critical role in life and consciousness. Physical systems construct biological forms, which can then in turn influence physical systems. Humans design the environment and at the same time the environment influences human behavior. In order to understand these systems, both upwards and downward driving factors must be simultaneously considered.

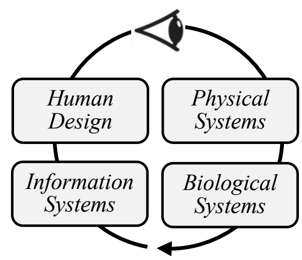


Figure 3-24 Feedback of Systems

Importantly, the notion that higher versus lower-level models are fundamentally separate and independent from one another, is not consistent with a unified view of science and nature. Lower-level theories and higher-level theories should reflect parallel descriptions of the same phenomena. The behavior of collections should be consistent with the parts, and parts should be consistent with collections. Bottom-up versus top-down factors is a product of how emergent models interact for a given interpretation. A unified view of nature entails that reality itself not delineated by the boundaries of lower-levels versus higher-levels.

Summary

This chapter provides a foundational understanding of emergence, which is when a higher-level theory arises from limiting the domain of applicability of a lower-level theory. The need for emergent theories is in part driven by the fact that nonlinear interactions of complex systems can create patterns where the whole is not predictable by the sum of the components. Emergent theories can also propose completely new measures, called heterostructural theories, to understand the world in terms of different quantities, like temperature or phases. Following the unity of science, emergent theories of nature should be consistent descriptions of one reality. In Part II – Theory, specific models used throughout the history of science will be reviewed to explore the patterns found in natural systems.

PART II - THEORY

Chapter 4 History

Example 4.1
Stonehenge

Ancient cultures built megalithic stone structures, utilizing systems of geometry, astronomy, and architecture.

Human history is punctuated by pivotal periods where societies have revolutionized scientific models and design practices. This chapter focuses on historical events where systems of science and technology have drastically transformed, providing insights into the development of civilization. An equation for this historical review can be written as when systems of the past (S_{Past}), have undergone changes and processes → to reach the present ($S_{Present}$). A historical review of impactful theoretical systems will contextualize modern science and current methods used to study natural patterns.

$$S_{Past} \rightarrow S_{Present}$$

Figure 4-1 History Equation

Societies across history have had fluctuating paradigms. Taking a simplified view, ancient and medieval philosophers often used holistic, systems-based, models of nature that emphasized relationships and connections. The success of Newtonian physics in the 18th century shifted Western science to prioritize a parts-based and predictable worldview, which dovetailed with industrialism, materialism, and resource extraction. In another shift of thinking, scientific evidence in the 20th century uncovered the importance of chaos, connectivity, and emergence. Moving into the 21st century, relations-based systems thinking is critical to coherently understand science and to address complex problems facing our world.

Paleolithic Technology

Human systems for understanding nature and designing tools reach far back into evolutionary history. The earliest evidence of humanoid technology are stone tools like choppers, hand axes, and projectile points, the oldest of which date back 3.3 million years.[43] The controlled use of fire is thought to have begun 1.5 million years ago. Fire served as a critical tool to make food more easily digestible and to enable travel to colder regions.[44] These early technologies showed that human ancestors were able to understand the environment and devise creative solutions to survive.

Humans continued to create more abstract systems of thought and practical technologies. Anatomically modern humans, *Homo sapiens*, arose around 200,000 years ago.[45] These early humans developed larger brains and new genes that would then support modern social traits, like complex vocal languages.[46] Language was an important system that supported human development because it was a method to pass down detailed information across generations outside of genetics and mimicking behaviors.[47] Language aided humanity's ability to organize into large societies, communicate ideas, and establish codes of conduct. Empowered by technology and language, humans migrated across Africa, Australia, Asia, Europe, and the Americas.

The development of symbolic art served as an important communication system used in prehistoric cultures. Paleolithic ("stone age") cave painting sites, like the Chauvet Cave in France dating to 30,000 BCE, features renditions of plants, animals, humans, and geometric symbols that may have served as an early pictograph language.[48] Another famous Paleolithic art piece is the Venus of Willendorf (24,000-22,000 BCE), a stone figurine thought to symbolize fertility and femininity.[49] This art shows that early humans were deeply observant of nature and utilized abstract symbols.

Neolithic societies could organize and solve complex engineering problems. One of the earliest examples of Neolithic architecture is Göbekli Tepe, a stone city complex in Turkey that may date back as far as 9600 BCE.[50] The site features large megalithic stones with detailed carvings of animal figures that were transported to the location. This site is hypothesized to have served as a social and religious hub for early hunter-gatherer societies.[51] Göbekli Tepe is also particularly interesting as this megalithic structure is much older than comparable sites found around the world, showing that humans could create complex buildings earlier than previously assumed in anthropology and archeology.

Agricultural Revolution

An enormous leap in the development of scientific, technological, and economic systems occurred during the Agricultural Revolution, beginning around 10,000 BCE.[52] During this time, nomadic hunter-gatherer societies began cultivating plants and raising animals. Fertile river deltas in regions like Mesopotamia, Egypt, India, and China became hotbeds of growth. Human diets included more agricultural products, and human physiology and genetics adapted accordingly to improve digestion of milk and starch-heavy foods.[53] The wide abundance of food sources allowed population explosions in permanent settlements. These large population centers organized into city-states with economic and political systems.

The Agricultural Revolution drove innovations in systems of language, math, and geometry. For example, by 3500 BCE the Sumerian culture used a phonetic writing method, maintained records of astronomical patterns, and created the 360-degree circle still in use today. Ancient Sumerians approximated mathematical pi *(π = 3.14…),* which relates the area of a circle to the radius, as well as found Pythagorean theorem solutions to right angle triangles, shown on Figure 4-2.[54] The Sumerians even developed formulas to find the volume of cubes, pyramids, and spheres.

Megalithic stone structures built during the Agricultural Revolution displayed impressive systems of engineering and astronomy. The Stonehenge complex in England (3000 BCE) is known for circles of massive stones that closely align to the sunrise and sunset on the shortest day of the year.[55] Ancient Egyptians constructed large and precise monuments, like the Great Pyramid of Giza (2500 BCE or older) which nearly perfectly aligns along true north. The Thoth Hill temple in Egypt (3000 BCE) accurately aligns to the helical rise of the star Sirius, which requires detailed astronomical knowledge.[56] These buildings demonstrate that ancient cultures had advanced systems of architecture, geometry, and astronomy.

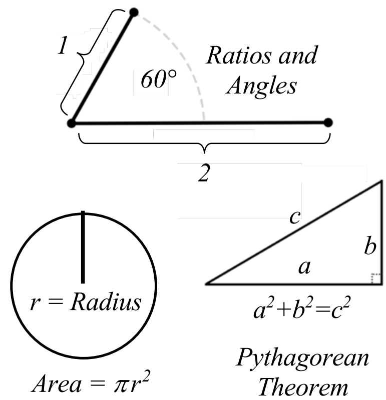


Figure 4-2 Ancient Geometry

Example 4.2 Giza Pyramid

Precise geometry, architecture, and astronomy was used to build the Great Pyramid of Giza.

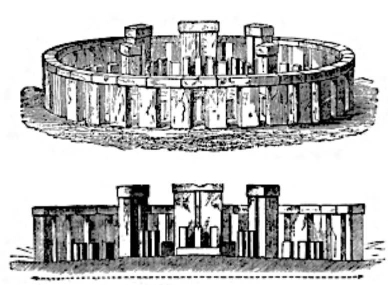


Figure 4-3 Stonehenge Complex

Ancient Frameworks

Ouroboros

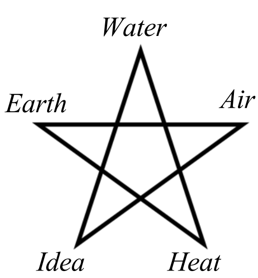
Endless Knot

Figure 4-4 Symbols of Change and Connection

Symbolic frameworks of nature from ancient cultures around the world often contained attributes of systems thinking, like cycles, feedback, and interdependency. For example, the Ouroboros symbol of a circular snake eating itself represents the cyclical nature of life and death as well as the changing seasons throughout the year. The Ouroboros is an ancient motif found in many regions across the globe, including Greece and India as well as the 14th century BCE funerary text of Egyptian King Tutankhamen.[57] This symbol conveys key aspects of systems thinking: the ability for feedback, reciprocal relationships, and a strange loop where the end feeds into the beginning.

The endless knot is another common ancient symbol found on clay tablets dating back to 2500 BCE in the Indus Valley.[58] This symbol has many interpretations, such as the intertwining and interactions of opposing forces leading to union and harmony. The symbol connects to a systems-based view by recognizing interconnected relationships of cause and effect and the ability for new patterns to arise from interwoven relationships.

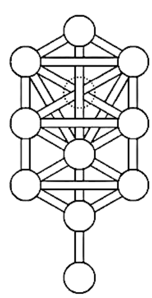
Hierarchical patterns, a common attribute of emergent systems, were symbolized in ancient models. Indigenous religions from the Americas, Asia, Africa, Europe, and Australia referred to higher- and lower-levels, connected by a cosmic tree.[59] The Tree of Life symbol in Jewish mysticism represents a hierarchical pattern spanning physical to spiritual dimensions. In Indian religions, the Sri Yantra was a symbol of a mystical mountain connecting nesting levels of reality. Hierarchies of levels are even embodied in the Hindu chakra system, which describes stacking energy centers in the human body, displayed on Figure 4-5.



Hygeia Pentagram

7. *Crown*
6. *Third-eye*
5. *Throat*
4. *Heart*
3. *Solar Plexus*
2. *Sacral*
1. *Root*

Chakra System



Tree of Life

Sri Yantra

Figure 4-5 Ancient Systems of Elements and Hierarchies

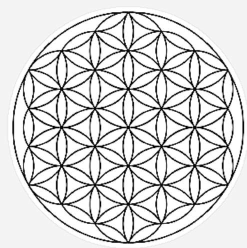
Ancient models of nature often emphasized the relationships between multiple elements, or phases, that could apply to a vast array of both internal and external phenomena. For example, Pythagoreans used a five element system, of earth, water, fire, heat, and idea, which formed the interlinked Hygeia, or health, pentagram.[60] This system was used to understand nature as well as personal health. Another system is the traditional Chinese Five Element theory of wood, fire, earth, metal, and water that is used in medicine and cosmology.[61] In the Chinese Five Element theory, energy continuously transforms through the different elemental phases and one imbalance can affect the entire system. Both the Pythagorean and Chinese Five Element systems emphasized connected relations that applied to the macrocosmic environment and microcosmic self.

Ancient Taoist philosophy utilized a model of nature's patterns that prioritized interdependence of opposing forces. In Taoist cosmology, nature is first an undifferentiated whole, called the Tao. From this void, opposing forces of yin and yang emerge. These opposing forces, like hot versus cold, or light versus dark, arise in connected pairs and enable the ability to define nature's characteristics. This model takes a systems-based view as individual elements are defined by relationships among the whole.

Taoist cosmology utilizes nesting iterations, a pattern common to systems. Permutations of yin and yang can recursively iterate, representing more complicated forces of nature. The Trigrams include three yin or yang elements, which creates eight permutations ($2^3 = 8$), each representing unique forces. Combinations of two trigrams form the sixty-four states ($2^6 = 64$) of the I Ching, which are used in divination to understand cycles in nature. From here, further permutations lead to the "10,000 things", or everything observed. Recursions that have a 2^n growth also occur in other systems, such as the states created from the number of n steps that can be true or false.

Example 4.3
Flower of Life

The Flower of Life is an ancient geometric symbol that has been found in Egypt, India, China, and Europe, with interlocking and nested patterns.



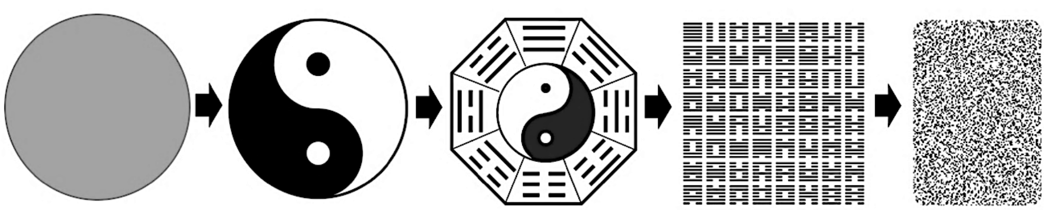
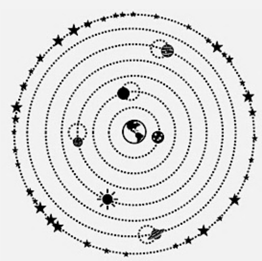


Figure 4-6 Taoist Cosmological System

Axial Age

Example 4.4
Ptolemaic Model

The standard view of the solar system in Europe in the 2nd century CE was a geocentric model where the Sun and planets revolve around the Earth in multiple nesting circles, called epicycles.



Order :

Center. Earth
1. Moon
2. Mercury
3. Venus
4. Sun
5. Mars
6. Jupiter
7. Saturn
8. Fixed Stars

New philosophical models and economic growth sprang forth in Eurasia from 800 BCE to 300 BCE. During this period, called the Axial Age, new schools of philosophical thought emerged across Greece, the Middle East, India, and China including Confucianism, Buddhism, Zoroastrianism, and others. [62] Written documents became more common during this time, helping to record history and transmit ideas. In a time of relative peace, a philosophical society could flourish and establish ideas that still have influence today. The Axial Age was also a time of economic growth. Trade expanded through routes like the Silk Road, which allowed ideas to disseminate across far distances.

During the Axial Age, mathematics became a more frequently utilized system for understanding nature's patterns. The 6th century BCE Greek philosopher Pythagoras was a foundational figure in mathematics and is credited with discovering that musical harmonies correspond to numerical ratios. This idea also led to the notion that the proportions of the heavenly bodies (Moon, Sun, planets, and stars) can be understood through cosmic ratios called the music of the spheres. The Pythagorean sect even correlated numbers with mystical meanings and believed ideas such as "God is number." [63] Pythagoras and other philosophers of this time pioneered the use of math to understand the patterns in nature.

In the 5th century BCE, the philosopher Democritus proposed that nature was composed of indivisible units of different shapes and sizes, called atoms, and that all of nature can be described by how these atoms combine. The notion of atoms later became one of the foundational pillars of a parts-based view of nature. While modern atomic physics shows that many of these ideas have relevance in certain domains, the concept of the atom was not pursued widely by other philosophers of Democritus' time, who instead favored relationship-based models of nature.

Ancient Greek philosophers often gave attention to geometry when searching for a rationale to describe nature. In *Timaeus*, dated 360 BC, Plato connects natural elements to geometric shapes called the Platonic Solids. The Platonic Solids, or regular solids, are 3-D polyhedrons made of triangles, squares, or pentagons that are identical from any face, edge, or vertex.

Plato associated the regular solids with the elements found in nature. The tetrahedron was associated with fire for being pointy and the cube was associated with earth for stacking easily. The octahedron was associated with the element of air for ease of sliding and the icosahedron was associated with water for ease of rolling, like small spheres. The dodecahedron, with 12 faces, was associated to the æther, which filled the region between the different celestial bodies and made the twelvefold division of the Zodiac in ancient Greek cosmology.[64] This presented a crude geometric model for understanding nature's patterns. Interestingly, modern physics later proved these shapes are essential for understanding molecular geometry and crystal lattices.

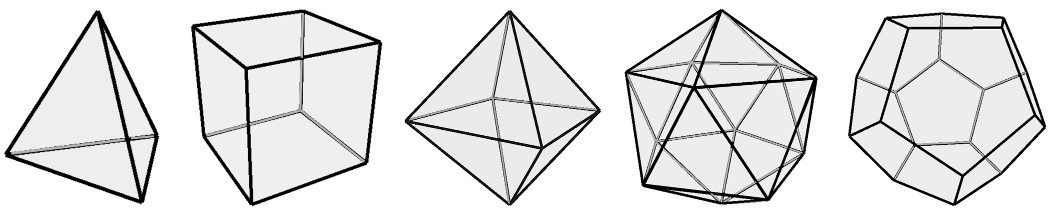


Figure 4-7 Platonic Solids and Associated Elements

Another advance in ancient Greece was the use of formal systems to deduce conclusions from a set of assumptions. Plato's student and colleague Aristotle, alive 384 BCE - 322 BCE, developed syllogistic logic which looked at statements like, "If Socrates is a man, and all men are mortal, then Socrates is mortal."[65] This statement would still make sense regardless of the terms, such as if *a* is equal to *b* and *b* is equal to *c,* then *a* is equal to *c.* By following the rules of logic, statements can be deduced as true or false. This laid the foundation for formal systems of logic and helped shape the scientific method. Euclid later used axiomatic and logical systems around 300 BCE to prove a variety of geometric theorems in the book *Elements*. The advances of ancient Greece helped establish the foundations for modern approaches of math, logic, and science.

Example 4.5 Euclid's Postulates

Euclid introduced a basic set of rules for geometry on a flat plane, which can be used to verify many geometric proofs. These postulates are:

1) *A line connects two points*
2) *Lines can be extended forever*
3) *Circles have a radius from a point*
4) *All right angles are equal*
5) *Parallel lines never intersect*

Expanding Empires

In the wake of the Axial Age, new empires transformed social and architectural systems. For example, Alexander the Great, leader of the Macedonian Empire of Greece, gained control of a vast region of land that included Persia and Egypt. The Greeks constructed massive architectural edifices utilizing stone columns, like the Parthenon temple, shown in Figure 4-8. The Roman Empire, beginning around 50 CE in what is present-day Italy, improved construction methods with inventions like Roman concrete that enabled blocks to be shaped to specification. The Roman Empire completed impressive infrastructure projects, including bridges, roads, aqueducts, dams, and amphitheaters, such as the Coliseum.[66] The Romans also pioneered new techniques for building arches and domes, such as the Pantheon temple, which remains the largest concrete dome ever built without reinforcing steel. These massive construction projects required advanced systems of math, architecture, and engineering, and helped set the stage for modern urban architecture.

Parthenon, Athens

Pantheon, Rome

Figure 4-8 Achievements in Ancient Architecture

Farther east, many new cultural and scientific developments occurred in the rise of the Islamic empires starting in the 7th century. The Islamic empires controlled a massive expanse of land from the Arabian Peninsula to North Africa, and managing this large empire required mathematics and detailed recordkeeping. The 8^{th}-13^{th} centuries marked the Islamic Golden Age when considerable scientific, cultural, and economic innovations took place. There was a heavy emphasis on translating past works, like Euclid's *Elements*, that were unavailable in Europe after the fall of the Roman Empire. Another important development included treating algebra as its own field of study that led to advancements beyond previous Greek geometers. The Islamic empires also progressed architecture with intricate geometric patterns, such as the Sheikh Lotfollah Mosque shown in Figure 4-9.

Example 4.6 Algebra

Al-Khwarizmi introduced algebra around 820 CE in a book that presented general solutions of linear equations, like: $ax+by=c$, and quadratic equations, like $ax^2+bx=c$.

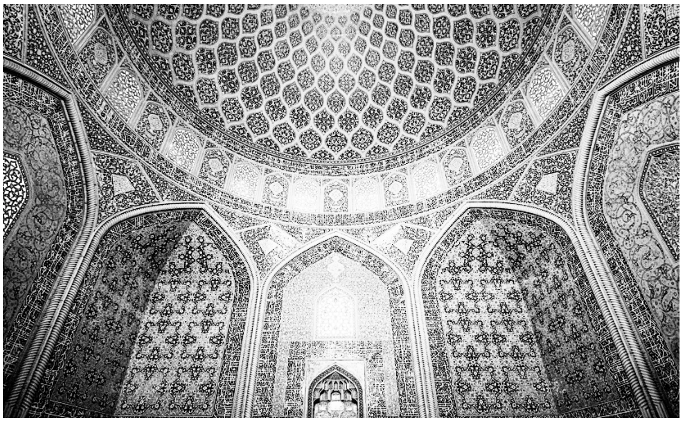


Figure 4-9 Islamic Architecture

Arabic mathematicians utilized (from earlier Indian sources) a positional base-ten number system to express large numbers, a method still used today. In this method, the digits represent a *0* to *9* value, and the position of the digit simultaneously represents what power of ten the value corresponds to, as displayed in Figure 4-10. Arabic mathematicians also introduced decimal point notation of fractions, where *0.001* corresponds to the ratio of *1/1000*.[67]

10-Digits: 0 1 2 3 4 5 6 7 8 9 *Positional: 2,526 = 2000 + 500 + 20 + 6*

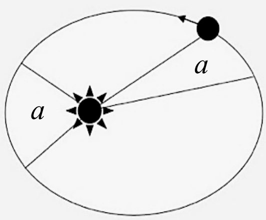
Figure 4-10 Hindu-Arabic Numeral System

Medieval Islamic scholars were able to explore both science and philosophy simultaneously in an integrated fashion. Science was not viewed as contradictory to religious philosophy, but rather seen as another way to understand the divine aspect of nature. This unity of science and philosophy allowed for both physical experimentation and the exploration of the limits of knowledge. Medieval Islamic scholars did not attempt to reduce all of nature and believed that some aspects of reality were beyond intellectual comprehension. Similarity, systems theory does not harshly distinguish between science and philosophy, and it uses both to understand nature.

The European Renaissance

Example 4.7 Kepler's Laws

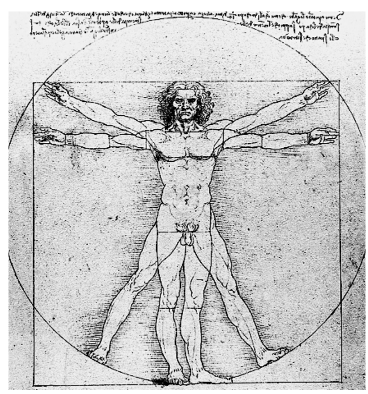
Johannes Kepler published rules of planetary motion in the early 1600's. These rules posited that the orbits of planets are ellipses (elongated circles) rather than nested circular orbits (epicycles). Additionally, a line segment joining a planet and the Sun sweeps out equals areas during equal time intervals, as shown below with area a. This law increased the ability to calculate planetary motion.



The European Renaissance was a revolutionary time of scientific, architectural, and artistic advancements centered in Italy in the 12th century. Europeans learned from the advances in Islamic sciences (in part due to the violent Crusades centered in the Middle East), picking up fields like algebra and trigonometry, as well as Greek manuscripts that had been absent during the medieval period, such as Euclid's *Elements.*[68] The reintroduction of these classics spurred Neoclassical architectural design and the desire to further build upon the scientific achievements of previous ancient cultures.

The European Renaissance was in full swing during the 15th and 16th centuries and famous polymaths such as Leonardo da Vinci, Michelangelo, and Raphael made notable contributions to science and art. Raphael's painting of the School of Athens depicts many ancient thinkers, demonstrating the large influence classical cultures had in the Renaissance. Another famous art piece was da Vinci's Vitruvian Man, which combined anatomy and geometry to display the symmetry of the human body. This drawing also finds an approximate solution to creating a circle and square with the same area, a common mathematical problem of antiquity.[69] Many of these polymaths, like da Vinci, took a systems view of nature, science, and art that highlighted connectivity. However, this was soon to change.

School of Athens by Raphael



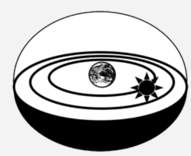
Vitruvian Man by Leonardo da Vinci

Figure 4-11 Renaissance Artworks

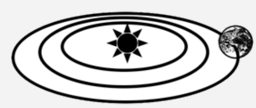
During this time, an extremely useful tool was created to study mathematical systems. The 17th century philosopher René Descartes proposed a coordinate grid of three axes (*x, y, z*) to model spatial relations. The Cartesian grid provides a way to ascribe algebraic meaning to geometric objects, and unites geometry and algebra into a single system. Descartes also increased the use of symbolic logic, like using the variable *x*, to represent mathematical and geometric relations.

Example 4.8 Evolving Cosmological Systems

Ancient Egyptians: Flat Earth with stars circling



Ancient Greeks: Circling geocentric shells



European Renaissance: Heliocentric orbits

A revolutionary theory was proposed in the 16th century that placed the Sun in the center of the solar system and challenged the status quo of a geocentric worldview, which is the idea that the planets and Sun revolve around the Earth. Nicolaus Copernicus was a proponent of a heliocentric theory and supported the Copernican Revolution, the view that the Sun is the center of the solar system. Galileo, armed with an advanced telescope, provided data to support the heliocentric theory. In the early 17th century, Johannes Kepler posed the idea that planetary orbits followed elliptical paths bound by specific laws of motion, which provided more evidence for the heliocentric model. In a heliocentric worldview, the Earth was no longer the center of the solar system, which challenged religious ideas of the time and set the scientific and religious thought communities at odds.

Isaac Newton provided the final breakthrough to mathematically describe the heliocentric theory. Newton famously contemplated why apples always fall downwards, and then had the revelation that the same force of gravity that made the apple fall also makes the Moon fall to Earth. Unlike the apple, the Moon never hits the Earth's surface due to its orbiting speed. Contrasting Aristotelian physics, which explains nature based on "causes" whereby objects serve different intrinsic purposes and goals, Newton's model used forces to calculate trajectories for objects in the solar system and on Earth alike.

Newton proposed that objects are attracted to each other by the force of gravity in proportion to their masses divided by the distance squared. Through this law of gravity, Newton was able to formulate a system of math equations that corresponded to astronomical observations. Using Descartes' coordinate grid and newly invented calculus, Newton proved that the planets must move in elliptical orbits and follow the relations Kepler had previously shown. This revelation finally addressed a question Plato had posed nearly 2000 years earlier: "How can the motions of the planets and stars be explained through mathematical laws?"[70]

A Clockwork Universe

A groundbreaking theory for modeling physical systems was proposed in 1687 by Isaac Newton in the book *Philosophiæ Naturalis Principia Mathematica*, which introduced three laws of motion that laid the foundation of classical physics, summarized below.[71]

1. *An object's motion does not change unless acted upon by a force*
2. *Force = Mass · Acceleration*
3. *Every force has an equal and opposite force*

Figure 4-12 Newton's Laws of Mechanics

These laws provided the basis for creating a mathematical theory of how objects are influenced through forces and collisions. This theory can very accurately predict some situations, like the trajectory of objects affected by gravitational force and the angles and forces of collisions, as shown in Figure 4-13. The 3rd law of motion, that "every force has an equation and opposite force," also aligns with the notion that energy and momentum are conserved. Newtonian physics began to reveal the powerful ability of math to accurately model systems of nature.

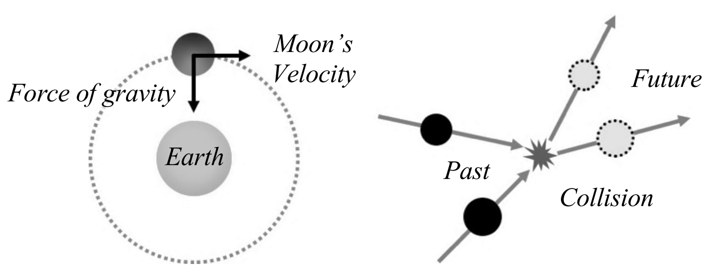


Figure 4-13 Force of Gravity and Collisions in Classical Mechanics

The success of Newtonian physics led many philosophers to believe all natural patterns are determined by predictable mechanisms. This concept, called determinism, is exemplified by the concept of Laplace's Demon, proposed by French scientist Pierre-Simon Laplace in 1814. This concept maintains that if a hypothetical being, or demon, knew the precise location and momentum of all objects, it could, with immense amounts of calculations, predict all future and past outcomes with full certainty.[72] The initial findings of Newtonian physics supported the case that all of nature was determined by solvable mathematics, and that Laplace's Demon could be theoretically valid.

The successes of Newtonian mechanics led the scientific community of the time to favor the belief that all questions about nature are predictable and decidable by mathematical laws, and that the properties of wholes can be fully understood by the models of the components. Accompanying a clockwork view of easy to calculate predictability, European scientists often assumed nature to be completely mechanistic and devoid of intelligence. This meant that the human body was seen as nothing more than a predictable machine of atomic processes with no connection to logic. Similarly, Cartesian duality held the view that the mind was a separate substance than matter. These philosophical stances pictured an unintelligent, mechanistic, and reductionistic world, which set the tone for scientific inquiry in the 18th and 19th centuries, prior to being challenged by discoveries of complex, chaotic, and informational systems.

Globalization and Sustainability

European colonialism radically changed global social and economic systems during and after the 15th century. Colonial empires, including the French, British, Portuguese, and Spanish, exploited societies in Africa, Asia, and the Americas and siphoned their natural resources at unprecedented levels. These colonial empires prioritized extractive economics, large-scale industry, the slave trade, and racial inequality, which all contributed to many of the injustices we face today.

For better or worse, the development of global empires increased the scale of impact and connectivity of humans across the planet. Global trade and travel became possible across Asia, Europe, Africa, and the Americas which spread new ideas and technologies. However, a more interconnected world meant greater risks, like military invasions, the spread of diseases, and greater potential for exploitation. These events showed that the world was becoming more connected, for better or for worse.

The Industrial Revolution in Europe sparked a series of new technologies that catalyzed productivity. Inventions of the 18th and 19th centuries such as steam engines, electric generators, and the automobile enabled a rapid expanse of cities, infrastructure, and large-scale agriculture. Moving into the 20th century, the shapes of cities shifted from circular plazas and footpaths into high density linear arrangements that were more easily accessible by automobiles. Strong steel frames supported the development of suspension bridges, skyscrapers, and huge radio towers that changed the skyline of cities to come.

Since the 1800s, the global population and its resource usage has dramatically expanded. It took over 200,000 years for the modern human population to reach one billion members, yet the human population reached 8 billion just 200 years later.[73] The UN estimates that population will plateau around 11 billion by 2100 if current fertility rates continue, as more developed countries usually have fewer numbers of children.[74] In the 21st century, the rapid resource usage of the human population is impacting the world at global scales. In an interesting twist of fate, the same tools that led to human expansion are now the potential sources of civilization's collapse.

Humans are now extracting resources and polluting the environment at global scales. Dwindling resource reserves resulting from overextraction exist in many sectors like cropland, fishing grounds, forest products, and grazing land. In 2022, the Global Footprint Network reported that it would take around 1.75 Earths to sustainably meet human demand at current levels.[75] Another unsustainable activity is the use of fossil fuels and other practices that result in the increase of carbon dioxide emissions—a primary driver of climate change. From 1800 to 2022, human-caused carbon emissions experienced over a 1000-fold increase, displayed in Figure 4-14.[76] The atmospheric carbon concentration is now higher than ever recorded in the past 800,000 years of ice core measurements.[77]

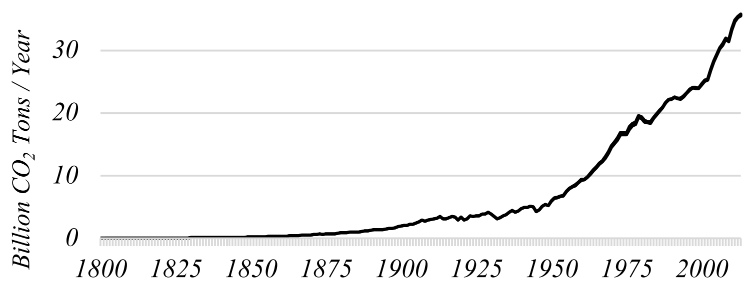


Figure 4-14 Global Human-caused Carbon Emissions

Extractive economic systems have been a large motivator of unsustainable practices and the exploitation of society and the environment. Capitalism often champions endless growth at the cost of resource depletion, and prioritizes profits over public gains, as summarized in Figure 4-15. These economic models increase the risks of socioeconomic crises and impede the ability to generate long-term sustainable solutions. Sustainability works to transform extractive economic models into regenerative economic models that promote longevity, renewable resources, and serves many social stakeholders.

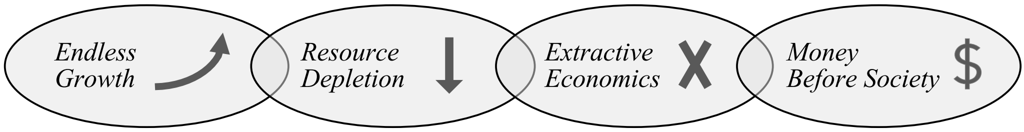


Figure 4-15 Unsustainable and Extractive Economics

The environmental movement began to arise in its modern form as a response to the negative environmental and social impacts of the 19th century Industrial Revolution. This movement has increased in the 20th and 21st century, there has been a growing efforts to redesign civilization's relationship with the environment for long-term sustainability. Inspirational books from the 1960s to 1970s brought environmental problems to the public's eye, like Rachel Carson's *Silent Spring* and Donella Meadows' *The Limits of Growth*. These books supported the view that human-caused pollution can negatively influence the environment and that there are limits to Earth's resources. Environmental science, which has since grown to a large field in the 20th and 21st centuries, highlights that human actions and nature must be considered together as an interacting system.

In the 21st century, sustainability has become a more general concept beyond environmentalism that includes economic and social systems. The sustainability movement has attracted public attention due to the increasing impacts of climate change, population growth, and resource constraints. The new science of sustainability shows that humans can impact the global environment, and that humanity needs to redesign our technology for long-term viability. Sustainable systems support resilient interconnectivity instead of isolated individualism, and the balance of cooperation and competition, instead of extractive competition. As a whole field, sustainability presents a new scientific revolution for learning how to harmoniously manage the environment, society, and economy—and systems thinking is core to this change.

Summary

Scientific and philosophical models have dramatically changed over time. Ancient worldviews skewed systems-based, but this changed in the 18th century in Europe with mechanistic science, which supported a reductionist and predictable model of nature. Going into the 21st century, there is a new revolution of systems science and sustainability that presents a view that nature and society is interrelated through numerous complex and connected relationships.

Chapter 5 Equilibrium

Example 5.1
Balancing Rocks

A system of rocks is able to maintain an equilibrium under the force of gravity if the weight is equally distributed around a balance point.

Equilibrium provides a common pattern and foundational scenario to study systems. A system in equilibrium has a set of quantities that does not change from the initial to the final state, written as $\Delta X = 0$, where $\Delta X = X_{Final} - X_{initial}$. In physics, X can represent measures like velocity and forces that remain consistent over time. For example, Newton's law that when one object exerts a force on another object, the second object exerts an equal and opposite on the first, means the total forces will sum to zero, written $\{Total\ Forces = 0\}$, and remain at equilibrium over time.

$$S:\{\Delta X = 0\}$$

Figure 5-1 Equation of Equilibrium

Equilibrium is integral to many fundamental concepts in physics such as the conservation of energy and the principle of least action (the tendency for systems to follow a path of least resistance). The principle of least action can be seen in the smooth curves of soap bubble films and has applications for efficient architectural designs. Many models used in modern physics, from electromagnetism to gravity, are solutions that minimize action and conserve quantities, like energy and momentum. Systems in equilibrium often have equations that are easy to predict and solve for. Systems at rest in equilibrium provides a conceptual steppingstone to understand more complex and dynamic scenarios.

Vector Fields

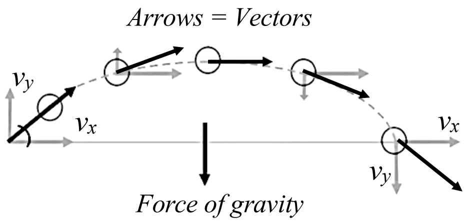


Figure 5-2 Velocity Vectors in Trajectory

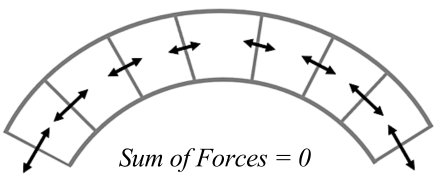
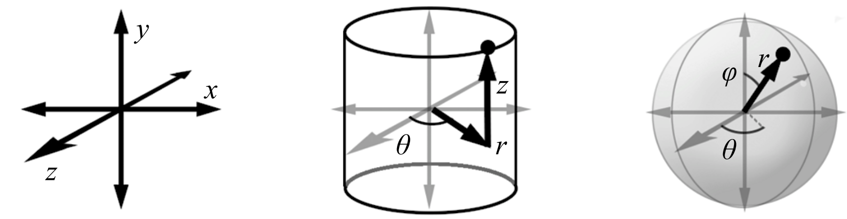


Figure 5-3 Arch Vectors in Equilibrium

A vector is a useful mathematical tool to model equilibrium or lack thereof. A vector is similar to a number in that it has a magnitude *(1, 2, 3, ...),* but is different because it also has a direction. Vectors can be broken down as the sum of multiple component vectors in a coordinate system, like the *x, y,* and *z* Cartesian axes. For example, a ball moving on a trajectory on a plane will have a vector along the *x* axis, representing horizontal velocity v_x, and another vector along the *y* axis, representing vertical velocity v_y, for a total velocity of $v = (v_x, v_y)$. In this example, the force of gravity decreases the velocity along the vertical axis, but has no effect on the horizontal velocity, creating a curved trajectory depicted in Figure 5-2.

A system will be at static equilibrium when all the force vectors balance and sum to zero $S : \{Forces = 0\}$.[78] For example, a stable arch has equal and opposite forces along each juncture to create a system that is structurally stable, shown in Figure 5-3. If the sum of forces did not equal zero, the arch would experience forces that cause acceleration, following Newton's law of $Force = Mass \cdot Acceleration$.

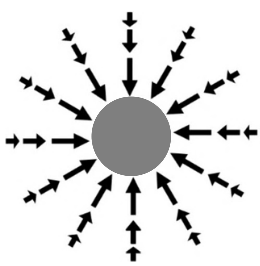
Vectors are defined within coordinate systems, such as the Cartesian grid of *x*, *y*, and *z* axes. An essential feature in a coordinate system is that each of the axes are positioned at right angles, or orthogonal, to one another. Orthogonality allows each axis to operate independently, which means vectors can be broken down into individual components that do not influence one another. For example, a change in the *x* axis does not influence the *y* or *z* axis. This also means that for a system to be in equilibrium, the sum of forces must equal zero along each independent axis. Other coordinate systems exist as well, such as cylindrical coordinates and spherical coordinates, which is comprised of a horizontal angle θ, vertical angle ϕ and radius *r*, as shown in Figure 5-4. Mathematical results will be identical regardless of the coordinate system chosen, but some coordinates make it easier to find solutions. For example, it is easiest to use spherical coordinates when modeling an atom or the Earth, due to the approximately spherical symmetry.



Cartesian Coordinates *Cylindrical Coordinates* *Spherical Coordinates*

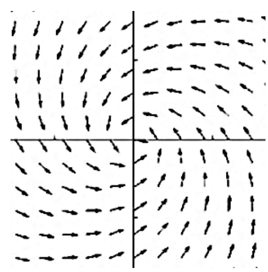
Figure 5-4 Orthogonal Coordinate Systems

A common method to model systems is by a vector field ***V***, which defines vectors—possessing magnitudes and directions—at each point in a continuous space. Boldface font is used to denote fields, like ***V***, that represent a continuous space of values. Earth's gravity can be modeled as a vector field, as shown in Figure 5-5. In this field, the vectors all point to the Earth's center of mass and have greater magnitude, signified by longer arrows, closer to Earth. A spinning vortex of water can also be modeled with a vector field. In this field, the vectors will have a greater spin closer to the center of the vortex. Vector fields provide a way to represent forces in a continuous space and calculate results, such as trajectories. Another simpler case of a field is a scalar field which has magnitude, but no direction at each point, such as a distribution of temperature values in a volume of air.



*Vector Field **V** of Gravity:*

Vectors point towards the center and decrease in magnitude closer to center



*Vector Field **V** of Vortex*

Vectors spiral around the center with a greater twist, closer to the center

Figure 5-5 Continuous Vector Fields

Vectors are used in interesting ways across disciplines to model systems. For example, vectors can be used in computer software and be applied to abstract spaces of data. Vector fields can even be used in more extravagant coordinate systems with additional dimensions, as is the case with spacetime in the theory of relativity. From physical to abstract cases, vector fields provide an indispensable tool to model systems.

Rates of Change

Example 5.2
Stocks and Flows

A system's stock changes by the flow rate. A stock in a bath is the water level, and the flow is the rate of how the water changes. A system is in equlibrium when the total flow is zero. The flow rate is found by the derivative of the stock over time, while the stock is found by the integral of the flow over time.

$$Stock = \int_0^t Flow$$

$$Flow = \frac{\partial Stock}{\partial t}$$

Rates of change can distinguish a system's quantities that are either changing or in equilibrium. Velocity v represents the rate of change of distance x over change in time t, written $v = \Delta x / \Delta t$. The change in the velocity over the change in time is called acceleration, written $a = \Delta v / \Delta t$. These rate changes represent averages over a time period, however, calculus introduces the derivative d to define the rate of change at each instant. For example, the derivative of distance over time provides the velocity at each instant, $v = dx / dt$.

The derivative of a function $f(x)$ with respect to x, which is written $df(x) / dx$, is the rate of change when the change approaches zero, $\Delta x \rightarrow 0$. The derivative is geometrically equivalent to the slope of the function, which is the change in height $(f(x+\Delta x) - f(x))$ divided by the change of width Δx, as shown in Figure 5-6. Integrals, written as $\int$, are the reverse operation of a derivative and are geometrically equivalent to the area covered by a function $f(x)$. For example, the integral of acceleration over time returns velocity, and the integral of velocity over time returns distance. Integrals can solve difficult problems, like the total distance traveled with a changing velocity, or the total area covered with a changing path.

$$\frac{df(x)}{dx} = \underset{\Delta x \rightarrow 0}{limit} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$$

$$Derivative = Infinitesimal\ Rate\ of\ Change$$

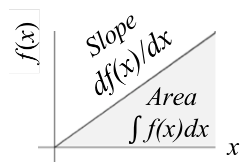
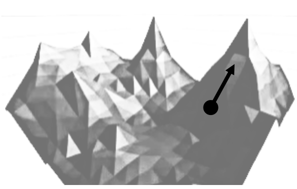


Figure 5-6 Derivatives and Integrals

Partial derivatives ∂ take the derivative in respect to one variable while holding others variables constant in a multivariable function, like $f(x, y, z)$. A useful tool is the del operator ∇, which is the partial derivative along each (x, y, z) vector component, following $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$. The del operator can be used to find the gradient, the total direction and magnitude of greatest increase of slope. For example, if a mountain surface $\boldsymbol{M} = f(x, y)$ defines the height at each point x and y, the gradient $\nabla \boldsymbol{M}$ points in the direction of steepest ascent.[79] The gradient can be used to find how water will flow down a hill, which will be opposite to the direction of steepest ascent. Gradients can also be used to find rates of change in volumes, like the direction where temperature changes most quickly in a volume of air.



$\boldsymbol{M} = f(x, y) = Mountain$

$\nabla \boldsymbol{M} = Mountain\ gradient$

$$\nabla \boldsymbol{M} = \frac{\partial \boldsymbol{M}}{\partial x} x + \frac{\partial \boldsymbol{M}}{\partial y} y$$

Figure 5-7 Surface Gradient

The del operator ∇ can produce other useful descriptions of a vector field ***V***, such as divergence and curl. The practical interpretation of these terms is that the divergence describes the flow in and out, and the curl correlates to the twist and rotation, associated with a source that is disrupting and altering the field.[80] Mathematically, divergence is calculated with the dot product $\nabla \cdot \boldsymbol{V}$ and the curl uses the cross product $\nabla \times \boldsymbol{V}$. A summary of the geometric interpretations of different rates of change is summarized below in Figure 5-8. These tools can model how multi-dimensional vector fields behave.

Name:	*Gradient*	*Divergence*	*Curl*
Notation:	$\nabla \boldsymbol{V}$	$\nabla \cdot \boldsymbol{V}$	$\nabla \times \boldsymbol{V}$
Meaning:	*Slope's direction*	*Flow in and out*	*Twist and rotation*

Figure 5-8 Interpreting Vector Fields

Divergence and curl arise in modeling electromagnetic fields. Electric fields are created by electric charges, like a positively charged proton or negatively charged electron. Like gravity, electric fields are proportional to the source magnitude divided by distanced squared, but electric fields can point toward or away from the source depending on charge. A stationary electric field ***E*** has a divergence proportional to the charge density $\boldsymbol{\rho}$, but zero curl. Magnetic fields ***B*** are created by moving charges, called currents, and have a curl proportional to the current density ***J***, but zero divergence. Maxwell's equations of static electric and magnetic fields are displayed in Figure 5-9 and show the stark difference of the in and out divergence of electric fields versus the twisting curl of magnetic fields.[81] A critical caveat is that dynamic, or changing, electric fields can induce magnetic fields and vice versa, and both can be expressed as a unified electromagnetic field.

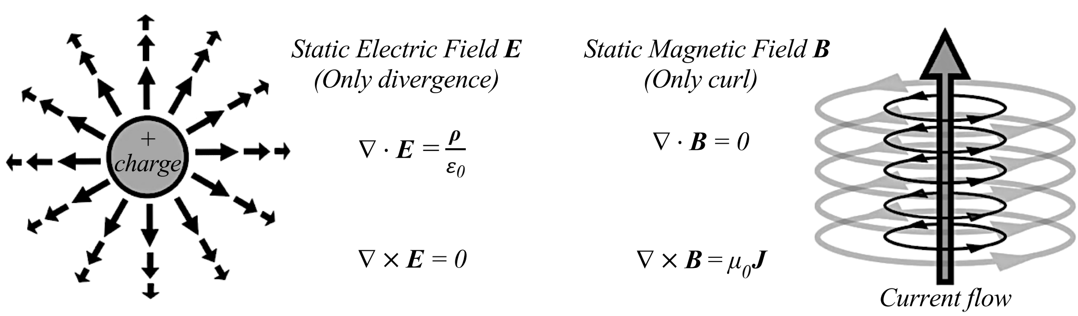


Figure 5-9 Divergence and Curl in Electric and Magnetic Fields

Laplace's Equation

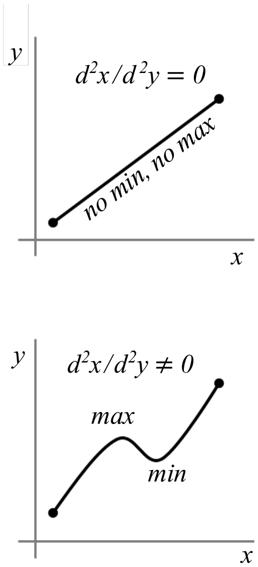


Figure 5-10 Second Order Derivative

Another method to study the behavior of systems and equilibrium is to analyze the second order derivative, written d^2, which is the "rate of change" of the "rate of change". When considering a path graphed on the *x* and *y* axes, the first-order derivative, dx / dy is the slope or rate of change of *x* with respect to *y*. The second order derivative, d^2x / d^2y, is the rate of change of the slope. A positive second order derivative means that the slope is increasing with an upward curve, and a negative second derivative means that the slope is decreasing with a downward curve. When the second order derivative is equal to zero, the slope is a constant straight line and does not increase or decrease, as shown in Figure 5-10.

Laplace's equation defines a field where the second order derivative equals zero, which is a critical tool for modeling systems in equilibrium. When the second order derivative is zero, there are no maximums or minimums inside the boundary points, and the end points are connected in the most efficient path, like a straight line as well as other optimal forms. Second ordered derivatives can be extended to three spatial dimensions of *x*, *y,* and *z* by squaring the del operator, written ∇^2. Laplace's equation is satisfied when the second order rate of change ∇^2 of a field $\boldsymbol{V}$ equals zero.[82]

Laplace's equation: $\nabla^2 \boldsymbol{V} = 0$ Del Squared $\nabla^2 = \left(\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2} \right)$

Figure 5-11 Laplace's Equation and Del Squared

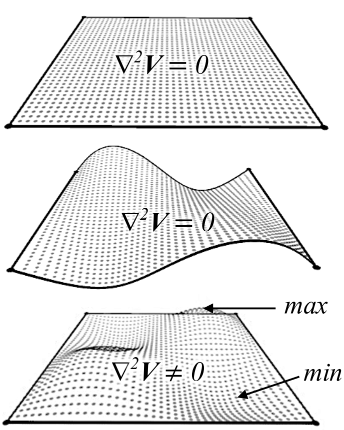


Figure 5-12 Laplacian Surfaces

Laplace's equation commonly comes about in systems that optimize surface area because this equation yields efficient geometries that avoid local minimums and maximums. For example, if a square ring of metal is dipped into soapy water, a film surface will form into a geometry that minimizes the surface area and follows Laplace's equation.[83] Even if the metal ring is bent, the soap film will form into a smooth surface that avoids local bumps and dips. It should be noted that soap films that enclose a region of space, such as spherical soap bubbles, do not follow Laplace's equations and can have a constant average curvature. However, all soap films still form in geometries that minimize the surface area to cover a given volume under the given forces.

In a static state of equilibrium, both the electric field and the gravitational field follow Laplace's equation between point sources of mass or charge.[84] A charged source in an electric field will create a local minimum or maximum in the field, but the field between these sources will form a perfectly smooth contour, similar to the smooth shape of a soap film. This remains true even with multiple sources in the same field. In Figure 5-13, the white and black dots represent positive and negative charges, and lines are drawn where the electric field has a constant force, like elevation lines on topographical maps. These field lines follow a Laplacian form, with no local minimums or maximums between point sources. Magnetic field lines are perpendicular to electric field lines, and form accompanying optimal forms.

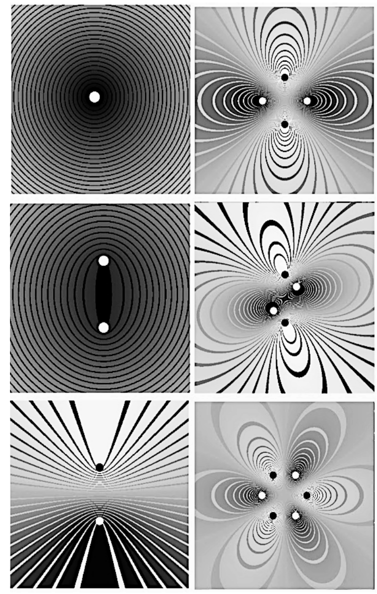
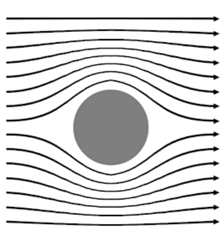
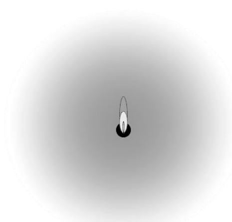


Figure 5-13 Laplacian Shapes in Electromagnetic Fields

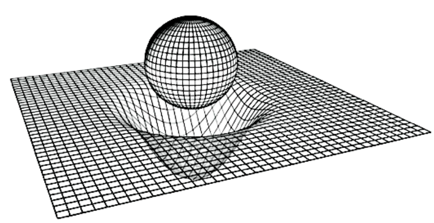
Many systems of nature modeled by vector fields settle into a smooth Laplacian contour in a state of equilibrium. For example, when tracing a steady flow of non-viscous water without turbulence around an object, the flow paths will create a smooth Laplacian distribution with no minimums or maximums. The heat distribution in a static system, such as a heat field around a steady candle, follows the Laplace shape. The air around the candle smoothly transitions from hotter (closer) to colder (more distant). While there may be small temperature fluctuations at the microscopic level, Laplace's equation provides an effective approximation to model the heat distribution in equilibrium. An important caveat is that Laplace's equation is broken when fields are not static, like with moving charges, masses, and heat fluxes, which create waves and disturbs the static equilibrium.



Non-Viscous Streamlines



Static Heat Field



Static Gravitational Field

Figure 5-14 Systems with Laplacian Fields

Equilibrium Forms

Stable systems in nature are maintained through balancing forces. In stable stars, for example, the inward forces of gravity must be opposed by the outward pressure of the star's molecules, to create hydrostatic equilibrium.[85] In the stable orbits of our solar system, a planet's velocity is just fast enough to create a centrifugal force equal and opposite to the gravitational force of the Sun. A lower speed would cause the planet to spiral into the Sun and higher speed would cause an outward spiral. At the atomic scales, helium atoms are highly stable, because two negatively charged electrons are electrically balanced with two positive protons. These two electrons completely fill the first wave orbit, creating an inert atom that does not tend to react. These stable systems balance the relevant forces and avoid undergoing change unless acted upon by external forces from the environment.

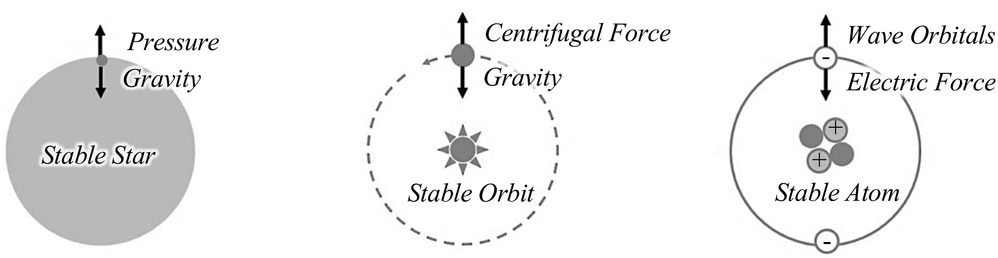


Figure 5-15 Stable Systems in Nature

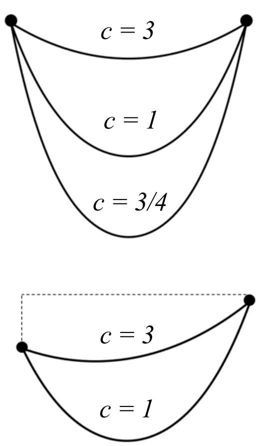
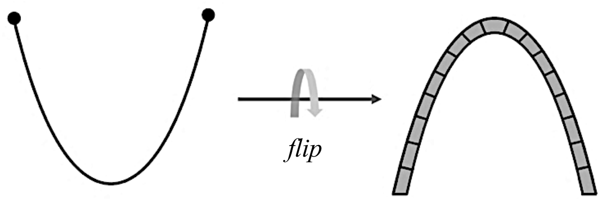


Figure 5-16
Catenary Curves

A common form of stable equilibrium is the catenary curve, which occurs as ropes or other string-like objects optimally balance the force of gravity when hanging. These curves are similar to a Laplacian shape, but they also have a degree of tautness marked by the constant *c* that creates a single dip. For example, a tighter rope is described with a larger *c*, and a looser rope has a smaller *c*, as shown in Figure 5-16. Catenary curves can be applied to find engineering solutions that efficiently balance forces, such as in suspension bridges.

Additionally, the inverse of a catenary curve can be used in architecture to optimally balance compression under gravity. The hanging catenary curve of a string is only held aloft by pure tension, the resistance to outward-pulling forces, and is not held up at all by compression, the resistance to inward-squeezing forces. Conversely, the flipped version of a catenary curve creates a form with pure compression and zero tension, a property that can be utilized in architecture.

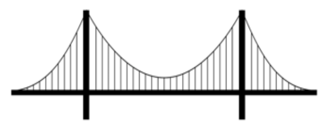


Catenary Curve: 100% Tension *Catenary Arch: 100% Compression*

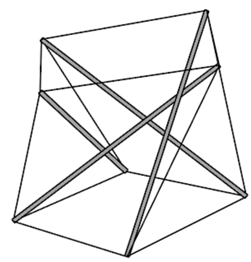
Figure 5-17 Catenary Arches

Catenary curves can be seen in Spanish architecture.[86] For example, a traditional method used in Spain to find optimal shapes of arches is to hang a rope between end points covering the same distance, then invert the curve. Antoni Gaudí, a Spanish architect in the late 1800s, built arches that followed catenary curves and used intricate hanging chain models to find efficient architectural geometries. The pinnacle of Gaudi's design, the Sagrada Família in Barcelona, features beautiful columns inspired by catenary curves and tree branch forms. These designs work to efficiently balance compression forces.

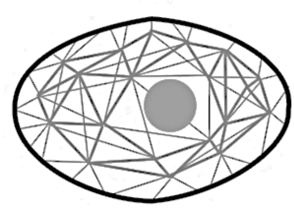
Equilibrium in a system can also be achieved through tensegrity, which is a combination of tension and compression forces. Tensegrity forms can have extremely high strength-to-weight ratios by utilizing lightweight materials with immense tension strength. For example, a lightweight string can suspend a large weight under tension. Suspension bridges utilize tensegrity to optimize strength-to-weight ratios and flexibility. Heavy beams support compression forces, while relatively light-weight metal cables hold the bridge up in tension. Tensegrity can be used in many geometric arrangements. Figure 5-18 displays a tensegrity form of four rods and twelve strings to support compression and tension forces.



Suspension Bridge



Rods and Strings



Cellular Cytoskeleton

Figure 5-18
Tensegrity Forms

Tensegrity helps explain the function of numerous biological systems. In the human body, the skeletal system supports compressive forces, while the tendons and muscles are connected through tension. This provides a flexible system with a high strength-to-weight ratio. At the cellular level, the cytoskeleton (a network of protein filaments and tubules that give cells their shape) is also suspected to utilize tensegrity.[87] Following these theories, the cytoskeleton's microtubules provide compression strength, while the filaments provide tension strength, enabling the cell to have a flexible, strong, and lightweight structure.

Principle of Least Action

One of the most powerful physical theories to study systems in motion is the principle of least action. This principle asserts that out of all possible trajectories of motion, the real physical trajectory will be the option that minimizes the action. For example, water will flow downhill and reduce the potential energy of gravity, not flow uphill. Similarly, a metal coiled spring will move toward the equilibrium, rather than an outstretched or compressed position.

Analyzing the units of action provides an intuition of what it means to minimize action. Action uses units of (*energy · time*) or (*mass · speed · distance*). So, the principle of least action is followed when energy is minimized over a given time interval or the distance is minimized for a mass moving at a given speed. For example, a moving object not exposed to force will travel in a straight line because that is the shortest path of least distance. Similarly, water under gravity will flow down a hill's gradient over time because it is the most time efficient way to lower potential energy.

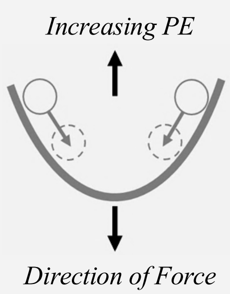
Action is defined as the integral over time of the Lagrange L, which is the difference of kinetic and potential energy. The simple equation that action is minimized over time can even be used to derive Newton's laws of motion, such as $F = ma$.[88] The principle of least action can be thought of as more fundamental than the equations of motion, as it dictates which equations of motion are valid. The principle of least action is deeply related to equilibrium, as action is minimized and zero over small finite changes δ.

Example 5.3
Force Orientation

Forces optimally lower potential energy by following a path of least resistance. A force F is inversely proportional to the change of potential energy PE.

$$F(x) = -\frac{\partial PE}{\partial x}$$

The potential energy of gravity increases with a greater height ($PE \propto height$) so the force of gravity points to reduce height. For example, a ball will roll toward to the lowest point to reduce PE.



Action is minimized in real trajectories (δAction = 0)

Action Units: (Mass · Speed · Distance), or (Energy · Time)

Action $= \int Ldt$ $\quad L = (Kinetic\ Energy - Potential\ Energy)$

Figure 5-19 Principle of Least Action

The principle of least action provides the backbone for generating equations of motion for many physical models, such as thermodynamics, electromagnetism, and fluid dynamics.[89] Every known fundamental force in physics, from general relativity to quantum mechanics, follows the principle of least action in some fashion. Each of the fields in the standard model of particle physics can be reformulated as Lagrange L with the shared property that action is minimized.[90]

The principle of least action combines with symmetry to produce another fundamental concept in physics, the conservation of energy and momentum. A symmetry assumed in physics and the unity of science, is that one set of universal rules are used to model nature regardless of the chosen coordinate system's spatial, angular, and temporal orientation. When the principle of least action is applied to these symmetries, it can be proven that interactions that are the same, or symmetrical, to a coordinate system of any location must conserve momentum. Interactions symmetrical to any angle must conserve angular momentum and interactions invariant to time must conserve energy. More generally, Emmy Noether proved in 1915 that every symmetry in a differential field, like translation, rotation, and time, has a corresponding conservation. [91]

Symmetry	*Principle of Least Action*	Conservation Law
Space-translation	→	*Linear Momentum*
Rotation-translation	→	*Angular Momentum*
Time-translation	→	*Total Energy*

Figure 5-20 Symmetry and Conservation Laws

Conservation laws can be used as a powerful tool by comparing a system at different points in time. In the interactions of particles, for example, the total energy and momentum in the initial set of particles must be equal to the total energy and momentum in the final set of particles, as shown in Figure 5-21. By starting with some known quantities, conservation laws can be used to find solutions to other unknown quantities. Many physics equations, such as the Schrödinger Equation in quantum mechanics, are formulated by comparing different expressions of total energy that follow conservation laws as well as the principle of least action.[92]

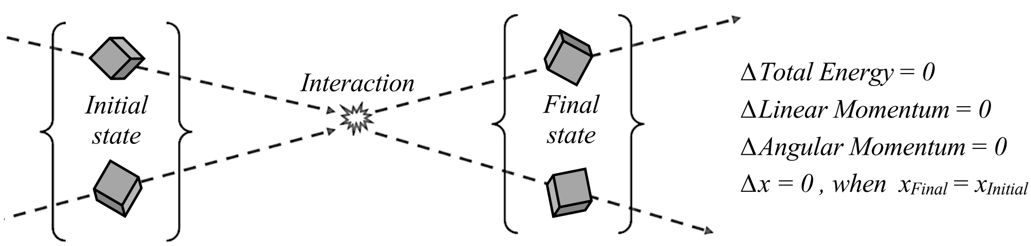


Figure 5-21 Conservation Laws and Interactions

Special Relativity

Example 5.4
No Static Ether

Prior to relativity, light was assumed to move at a constant speed compared to ether, the stationary space of the universe. Relativity showed that there is no stationary ether and that speed should rather be defined by the relationships between frames of reference. Speed is a relations-based property that is relative to each frame of reference, not absolute.

Proposed by Albert Einstein in 1905, relativity modeled a new symmetry and equilibrium of space and time, that the speed of light remains constant for all reference frames. Relativity was motivated out of the puzzling result that Maxwell's equations predict that light always moves at a constant speed regardless of the frame of reference.[93] Relativity solved these problems by posing a new system of space and time that allows for the speed of light c to always remain constant $\Delta c = 0$, even if a frame of reference is moving. This seemingly innocent equilibrium, that the speed of light is constant in all reference frames, required reforming concepts of space and time.

Relativity differs dramatically from classical physics. In Newtonian physics, light emitting from a car's headlights would equal the combination of the car's velocity v with the speed of light $(v + c)$. Conversely, relativity requires light to travel at the same speed when the car is both stationary and moving, contradicting Newtonian mechanics. At low speeds, this does not make a big difference, but when the car's velocity is close to the speed of light, the behavior is radically different. Relativity reconciles these differences with relativistic dilation, where measurements of distances, time, and mass can change depending on the velocity of the frame of reference.

To understand dilation, consider a clock with time units determined by how long it takes light to move from the bottom to the top of a box. When observing this clock in motion along the horizontal axis, the total observed time for the light to reach the top is defined as Δt and covers the distance $c\Delta t$, which includes the horizontal distance $v\Delta t$. In comparison, The dilated time perceived by the moving clock in its rest frame is defined as $\Delta \bar{t}$ and covers the smaller distance $c\Delta \bar{t}$, with no horizontal component. Equations comparing the observed time Δt to the dilated time of the clock in motion $\Delta \bar{t}$ can be solved using the Pythagorean theorem. The results show that time is slowed, or dilated, when the moving reference clock increases velocity.

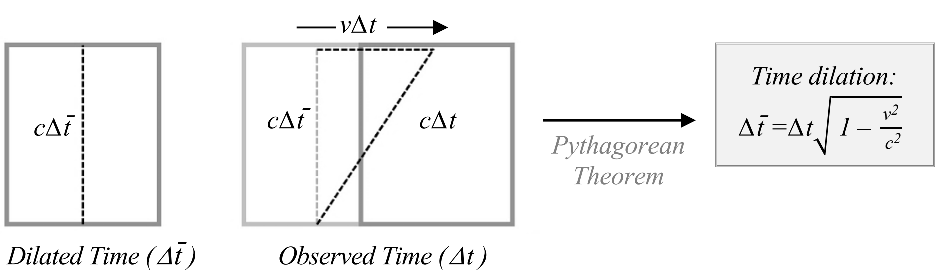


Figure 5-22 Time Dilation in Special Relativity

Time dilation is not merely theoretical, and experiments have shown that increasing velocities can slow the flow of time, such as slower decay times of particles moving with high velocities.[94] Similar proofs can be made to show that movement also dilates length. For example, a 12-inch ruler moving near the speed of light is shorter than a stationary 12-inch ruler. Another result of relativity is that increasing speed increases mass. The increase of mass means it is impossible to move faster than the speed of light because it requires more and more energy to move an object that becomes increasingly massive. Relativistic dilation shows that time, distance, and mass are not fixed and instead change between reference frames in motion. Relativity dismantled the notion that space, matter, and time are fixed and absolute, and put forth a new system of a relative world.

Relativity utilizes a 4-D coordinate system, called Minkowski spacetime that includes three spatial dimensions and a time component (x, y, z, ct). Time is multiplied by the speed of light to create units comparable with distance. The Minkowski spacetime coordinate system can more easily account for relativistic dilation and allows the laws of physics to remain consistent for frames in motion. In relativity, classical descriptions of length, time, mass, and energy are approximations. Instead, relativity expresses conservation laws and quantities that remain in equilibrium with four-vectors, with three units of space and one of time. Four-vectors drastically simplify electrodynamics and can express all Maxwell's equations in a single compact form.

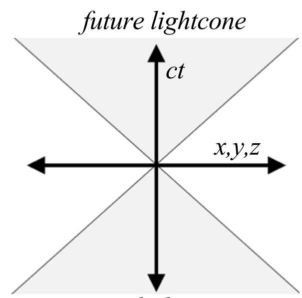


Figure 5-23
Lightcone Diagrams

Relativity commonly graphs spacetime with lightcone diagrams that draw space on one axis and time on another. A stationary object would stay in the same place on the horizontal axis but move up the vertical axis through time. In contrast, a beam of light would move at a 45-degree angle, traveling distance ct over time t. In a lightcone diagram, such as that shown in Figure 5-23, it is important to note that three spatial axes of (x, y, z) are being reduced to one axis. This means every horizontal slice is a volume of space at a given time. Lightcone diagrams are particularly useful to specify the limits of electromagnetic transfer between points in spacetime. For example, a location 15 light-years away from the origin point would require a minimum of 15 years to transfer information to it. The past lightcone defines the farthest distance that can send an electromagnetic signal to be received at an origin point over a given amount of time. The future lightcone determines the farthest distance that can receive information sent from an origin point within a given amount of time.

Curved Spacetime

Einstein's subsequent theory, general relativity, extended relativity to accelerating reference frames of reference, including gravity. Einstein's field equations propose that spacetime is curved by energy and mass which causes the apparent force of gravity. To explain curved spacetime, it is useful to first look at curvature in a 2-D Euclidean plane. Within a flat Euclidean plane, the sides of a triangle are straight, and the interior angles sum to 180 degrees. In a positive curvature plane, the lines bow outwards, and the angles sum to more than 180 degrees. In negative curvature, the lines bow inwards. In general relativity, mass causes the curvature of spacetime, and bends trajectories toward mass and energy sources.

Example 5.5
Einstein's Equation

General relativity utilizes Einstein's field equations to measure how mass and energy curves spacetime. This equation uses tensors, which expand vectors to a matrix with μ columns and ν rows.

$$\boldsymbol{G}_{\mu\nu} + \Lambda \boldsymbol{g}_{\mu\nu} = \kappa \boldsymbol{T}_{\mu\nu}$$

$\boldsymbol{G}_{\mu\nu}$ is the spacetime curvature, which is proportional to $\boldsymbol{T}_{\mu\nu}$, the mass, energy, stress tensor, as well as the constant κ. The term $\Lambda \boldsymbol{g}_{\mu\nu}$ is the cosmological constant, the curvature of empty space, which is expected to be negative to account for dark energy.

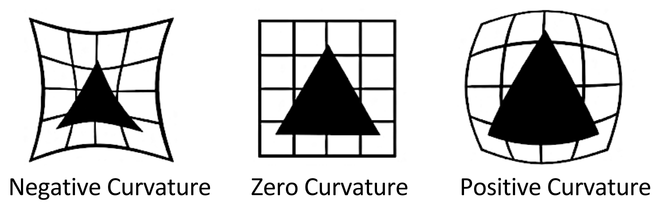


Figure 5-24 Curved Space

Trajectories in spacetime follow optimal paths of least action. Without gravity, an object in motion will travel in a straight line, the shortest distance between two points. With gravity, the optimal path follows a curved trajectory called a geodesic. Geodesics are the shortest paths through curvilinear spacetime and only appear curved in a flat space. We see an example of curved trajectories in airplane flight paths, which follow a straight line over a spherical globe but appear curved on a flat map projection. Another analogy of gravity is a heavy ball on an elastic sheet. When a small ball is on an elastic sheet, it will be attracted to a large center ball due to the curvature of the sheet imposed by the spheres, as seen in Figure 5-25. Gravity similarly curves paths by distorting spacetime.

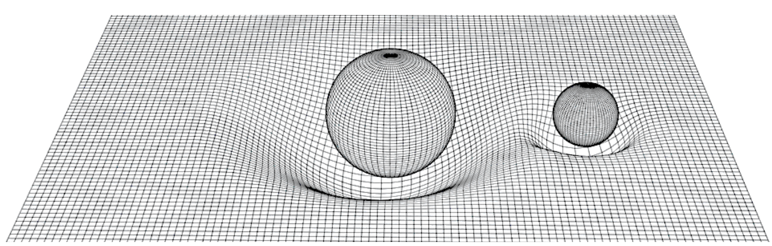
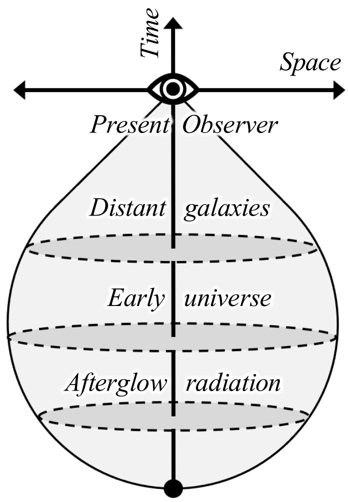


Figure 5-25 Curved Spacetime

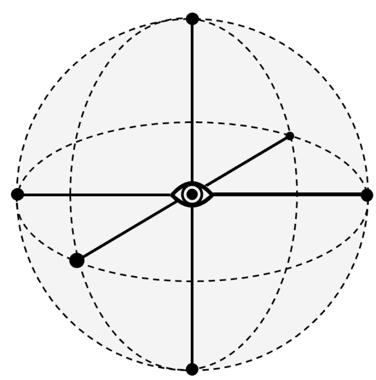
General relativity allowed for significant advancements in modeling the shape of the universe. Einstein's field equations contain a universal constant that defines the overall curvature of the universe, which was believed to be zero using available data at the time. After Edwin Hubble's discovery in the 1920s that distant galaxies are moving away from one another, it is now believed that the universe is expanding via dark energy, which correlates to a non-zero cosmological curvature. Reversing the expanding universe model into the past also justifies the Big Bang theory, which postulates that the universe originated from a single origin approximately 13.7 billion years ago.

An expanding universe means the universe was smaller in the past, which creates a curved lightcone of the past. Light observed from 10 billion light-years away shows the universe 10 billion years ago, which was smaller. This creates a pear-like lightcone in which observing farther into space eventually curves to the Big Bang singularity, as shown in Figure 5-26. Additionally, in every direction of observation—left, right, forward, backward, up, or down—farther distances will always curve back to the Big Bang. To provide a simplified analogy, moving in a straight line from any direction at the top of a sphere will always lead to the same point at the bottom of the sphere.

Another aspect of spacetime is that each reference frame is the center of its observable universe. Regardless of the location of the origin point of a frame of reference, the lightcone into the past will expand evenly in all directions and creates the effect that each observer is in the center of their observable universe. The analogy of a sphere can be used again as each location of a sphere's surface can be perceived as the top and center from its vantage point, with the horizon evenly curving away. Additionally, due to dark energy's negative curvature and the expansion of space, the observable universe is currently estimated to have a radius of 46.5 billion light-years, which is larger than how far light could normally travel in the 13.7 billion year age of the universe. Relativity presents a new geometry of cosmology that drastically transforms Euclidean notions of space.



Big Bang
13.7 Billion years ago



Present observable universe
46.5 Billion light-year radius

Figure 5-26 Lightcone in Expanding Universe

Universal Equilibrium

Equilibrium coincides with the core tenant of physics, that the universe conserves certain quantities, like energy.[95] Following the notion that the universe is isolated and all that there is, conserved quantities like energy, should not enter or leave, only change forms. Conserved quantities in an isolated universe U should be fixed and remain in equilibrium, written $\Delta U = 0$. Another implication of an isolated universe is a relationship between the a subsystem X and everything not contained in the system $(U - X)$. If a subsystem is closed and does not change conserved measures $\Delta X = 0$, everything outside of X must also not change these conserved quantities. Conversely, if a system is open to change $\Delta X \neq 0$, everything outside must also be open to maintain $\Delta U = 0$. If conserved universal quantities were violated $\Delta U \neq 0$, the properties of closed versus open subsystems would not be guaranteed.

Example 5.6
Conservation Laws

The universe is expected to conserve key measures relating to the total content and energy such as:

Conserved

Total Energy
Linear Momentum
Angular Momentum
Quantum Information

However, measures relating the order and organization of this content can change and are not conserved.

Not Conserved

Entropy & Order
Complexity Measures
Classical Information

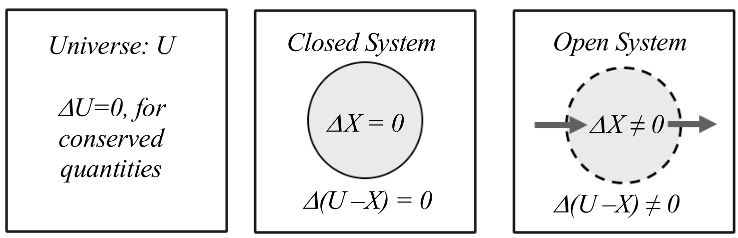


Figure 5-27 Universe with Closed and Open Systems

In modern physical theories the universe is closed to, and conserves, some quantities and is open to change in others. The universe is expected to conserve total energy, linear momentum, angular momentum, electric charge, quantum information, and the color charge in the strong nuclear force between subatomic quark particles. While mass is conserved in classical physics, following modern physics, like $E = mc^2$, it is possible for energy to turn directly into mass, changing the total mass. Furthermore, the weak isospin in the weak nuclear force, spatial inversion symmetries called parity, and types of quarks called "flavors", are often conserved, but can change in certain high-energy processes. Other measures are almost never conserved. Thermodynamic entropy tends to increase over time in isolated systems and can also be reduced in open systems to increase order and concentrate energy. Classical (non-quantum) information often increases, degrades, or even evolves over time. The complexity, relating to the difficulty to predict states, is also open to change over time.

Continuum mechanics is a powerful tool to express how conserved quantities, like energy or charge, move continuously across volumes. The continuum equations state that the change of the quantity inside a boundary plus the flux of the quantity over a boundary equals zero, as shown in Figure 5-28. For example, when the density inside a volume decreases, there must be a positive outflow of flux. In three dimensions, the divergence $(\nabla\cdot)$ measures the total inflow versus outflow and can be used to express continuity as $\partial(Density)/\partial t + \nabla\cdot(Density\ Flow) = 0.$[96] Many equations in physics, from fluid mechanics to electromagnetism, reformulate the continuum equation to derive equations of motion. These equations all stem from the notion that content can only change within a boundary if there's is a flux—an idea that follows naturally from an isolated universe.

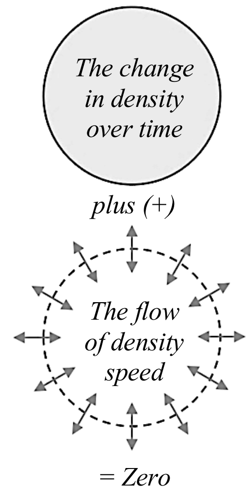


Figure 5-28
Continuum Equations

Universal equilibrium even plays a role in logic. The set of possible values, called the universal set U, or domain of discourse, should remain in equilibrium. Having a closed domain of discourse, $\Delta U = 0$, is necessary to ensure statements remain valid in a formal system. If members of the domain are free to be added or removed, then the truth values of a proposition p or negation *not* $p = \{U - p\}$ are not guaranteed. Equilibrium also relates to logical equivalence, and the difference of equal terms is always zero. For example, if $a = b$ is true, then it is also true that $a - b = 0$. Equalities should also remain stable in a formal system. This does not mean that every logical system is consistent, complete, or free of paradoxes, but that the expression of equality *"="* should not change.

Even in models where the physical universe and logical domains are closed, there is an extreme variety and potential for complexity in open subsystems. Fluxes of energy and matter across subregions can support high levels of change and organization, as seen in living systems. A closed physical universe or logical domain does not limit the complexity of any particular subsystem within it, but instead ensures that the conserved quantities remain at equilibrium.

Summary

Equilibrium, the principle of least action, and conservation laws are critical tools for analyzing physical systems. These principles are essential for generating models to describe electrodynamic fields, gravitation, thermodynamics, fluid dynamics, quantum probability fields, and relativistic spacetime. Equilibrium also plays a role in understanding models of the physical universe and logical domains. Subsequent chapters will explore systems that are not in equilibrium.

Chapter 6 Flux

Example 6.1
Splash of Water

Systems in nature are continually in motion. A flux disturbs a state of equilibrium, like a splash in a steady pool.

It is often said that change is the only constant. Models of equilibrium, where no force or energy is exchanged, only provides a small piece of the larger, dynamical, picture. Compared to systems at rest, the patterns resulting from systems in flux can be much more intricate and complex. Modeling systems of flux can bring insight to natural phenomena like light, sound, and the microscopic vibrations within matter itself. This chapter will explore common patterns of flux including growth, diffusion, waves, and nonlinear dynamics. More generally, these examples are part of a class of systems where the change of a quantity ΔX does not equal zero and is not in equilibrium, shown in Figure 6-1.

$$S:\{\Delta X \neq 0\}$$

Figure 6-1 Equation of Flux

Flux is essential to understanding systems, connectivity, and complexity. Systems of flux, and particularly those that are nonlinear, can generate chaotic unpredictability and complex patterns. This can be seen in open thermodynamic systems, like convection patterns, tornados, cyclones, and other forms that would dissipate without energy inputs. Living systems are another particularly complex system that requires an open flux of matter and energy to maintain organization and homeostasis. Highly complex systems can be enabled by the open exchange of energy, mass, and information.

Types of Flux

Systems of flux fall into broad categories. One category is linear change that grows or decays at a constant rate, or progressively moves towards an equilibrium. An example of a linear flux is the increase of entropy, which is the steady tendency for energy to disperse and order to decrease over time. Another category is cyclical fluxes, like a pendulum swinging back and forth. These oscillating fluxes are often modeled with waves. It is also possible for nonlinear and semi-cyclical patterns to form within an open flow of energy. The nonlinear patterns of a vortex or a tornado, for example, are highly complex and cannot be explained in linear terms. These broad classes of change in systems, summarized in Figure 6-2, will be explained further throughout the chapter.

Example 6.2
Flux in Open and Closed Systems

Flux can occur in closed and open systems depending if a measure is conserved X_C or not conserved X_{NC}. In an open system, both X_C and X_{NC} can be in flux. In a closed system only X_{NC} can be in flux. For example, a system closed to total energy can change kinetic energy if the potential energy changes.

$\Delta Energy_{Total} = 0$
$\Delta Energy_{Kinetic} \neq 0$
$\Delta Energy_{Potentail} \neq 0$

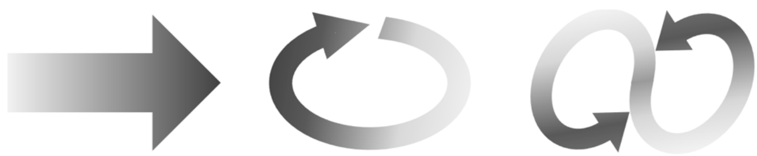


Figure 6-2 Types of Flux

Dynamic equilibrium is another interesting type of flux, which is created when a continuous change enables a stable form. Chemical reactions exist in dynamic equilibrium when the rate of a forward reaction is equal and balanced to the reverse reaction. For example, molecules in a glass of water are in constant flux, splitting into hydrogen hydroxide ions ($H_2O \rightarrow H^+ + OH^-$) and recombining ($H^+ + OH^- \rightarrow H_2O$). When the rates of splitting and recombining are equivalent, the liquid water will maintain a stable average chemistry, even though reactions are continuously occurring. Thus, a dynamic equilibrium is achieved by through a balance of an influx and outflux.

Dynamic equilibrium is a core concept for physical and biological systems to maintain themselves within a constant flux. In living systems, the ability to maintain homeostasis and biological structure is only achieved through the delicate balance of water, food, oxygen, and trace minerals. Dynamic equilibrium also plays a critical role in sustainable systems. If a system extracts too many resources it can deplete the environment, but extracting too few resources will not sustain the system itself. The goal to maintain a dynamic equilibrium is a common feature of systems across many disciplines.

Growth and Decay

Growth and decay are patterns of dynamic systems that arise when the flux rate is proportional to the amount of content. This relationship can be expressed as a formula where the change of x over time, dx / dt, is equal to the amount of x multiple by a constant, written $dx / dt = kx$. When the constant k is greater than zero, the system will grow, and if the constant is less than zero, it will decay, as displayed in Figure 6-3. The growth equation can be expressed through the exponential term ($e \approx 2.718...$), in the form $x_t = x_0e^{kt}$, which is why this formula is also referred to as exponential growth.

The growth and decay equation can model many systems, such as how the value of an investment account will increase as the result of continuously compounding interest. A greater quantity of money, or x, leads to greater returns for a given rate. The growth equation can also be used to model how a population of x grows with a constant rate of change. A greater population of x leads to more growth. Decay (negative k values) works in the opposite way: a given quantity will decrease and approach zero over time.

A growth and decay relationship can be combined to create a balancing equation similar to self-regulating populations of prey and predators. The predator-prey equations, that are further detailed in Figure 6-3, provides a model of balancing interactions between a population of prey as x and predators as y. When there are more prey x, there are more opportunities to feed more predators, but when there are more predators y, there are more agents to reduce the number of prey. This equation results in oscillations where the two population sizes balance with one another over time.[97]

While ecological systems do resemble some patterns of the balancing, growth, and decay equations, there are limitations. In realistic scenarios, chaotic events and the interactions of many species mean that these models are insufficient. With that said, these equations show how simple growth and decay forces can synergistically balance in a collective ecosystem.

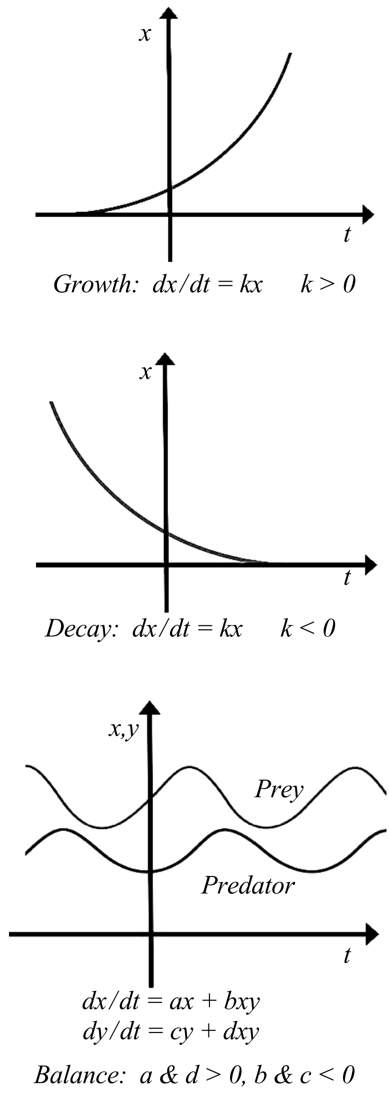
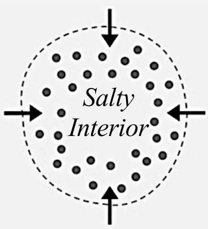


Figure 6-3 Growth, Decay and Balance Equation for Systems

Diffusion

Example 6.3
Diffusion in Biology

Diffusion creates a pressure gradient when different concentrations come into contact along a porous boundary. Diffusion pressure is used by cells to regulate chemical concentrations across membranes. For example, a cell with salty internal water will have a pressure to absorb outside fresh water.



Diffusion is a type of flux that describes many natural systems, such as the spreading of gases, liquids, or heat into a surrounding space.[98] Diffusion can be seen when concentrated molecules of dye pigment in a glass of water disperse over time until there is a uniform distribution, as shown in Figure 6-4. Similarly, heated air or a spray of perfume will tend to distribute itself throughout a room. The movement pattern of diffusion is for a substance to spread in all directions evenly, unless influenced by other forces. Over time, diffusion eventually settles into a dynamic equilibrium, where the average collective concentration is uniform, even though the individual particles are in random motion.

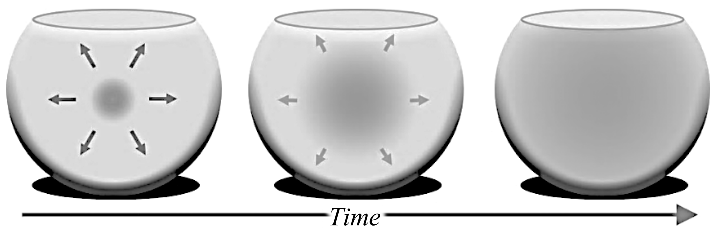


Figure 6-4 Diffusion of Dye in Bowl of Water

Diffusion can be modeled with a field V that defines the magnitude of a concentration at each x, y, z point. While random collisions can create microscopic variations, these smooth fields can serve as emergent models for macroscopic averages. In the diffusion equation, shown in Figure 6-5, the rate of change over time ($\partial V/\partial t$), or speed of the field, is proportional to the second order derivative over space ($\nabla^2 V$), which is geometrically the sharpness of local maximums. Sharp changes to concentrations spread apart faster, while smoother concentrations move more slowly. The diffusion coefficient α describes how fast a medium diffuses, which can be influenced by temperature, viscosity, and other factors.[99] As the system approaches uniformity, the rate of diffusion approaches zero and the concentration reaches a state of equilibrium. When the change over time ($\partial V/\partial t$) equals zero, the diffusion equation then simplifies into the Laplacian $\nabla^2 V = 0$.

$$\text{(Speed of Field)} \quad \partial V/\partial t = \alpha \nabla^2 V \quad \text{(Sharpness of Local Max)}$$

Figure 6-5 Diffusion Equation

Waves and Harmonics

Waves describe systems that oscillate, or repeat, over time. The most basic wave is simple harmonic motion, which is a steady back and forth movement. Waves can be modeled with the restoring force equation where the force F is inversely proportional to the displacement from equilibrium, written $F \propto -x$.[100] A swinging pendulum with a resting location at $x = 0$ provides an example of simple harmonic motion. When the pendulum moves to the right ($x > 0$) there will be a force in the left direction ($F_x < 0$), and when the pendulum moves to the left ($x < 0$) there will be a force in the right direction ($F_x > 0$). This restoring force creates a wave, which has a specific wavelength (distance from peak to peak), amplitude (height of a peak), and frequency (rate of oscillation). Solutions to the wave equation include the trigonometric functions of sine, cosine, and exponentials.

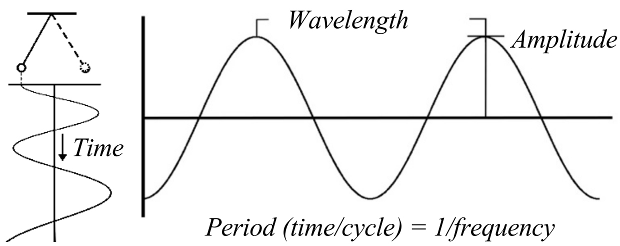


Figure 6-6 Waves, Pendulum, and Spiral

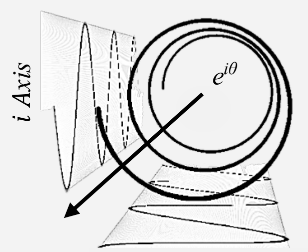
The wave equation can also be represented in a differential form, with a field $\boldsymbol{V}$ in the x, y, and z axes. In contrast to diffusion, where the first order derivative ($\partial \boldsymbol{V}/\partial t$), or speed of the field, is proportional to the sharpness of the concentration ($\nabla^2 \boldsymbol{V}$). In the wave equation, it is the second derivative ($\partial \boldsymbol{V}/\partial t^2$), or acceleration of field, that is proportional to the sharpness of concentration ($\nabla^2 \boldsymbol{V}$). This creates the effect that a concentration will oscillate around an equilibrium, following $F \propto -x$. The wave equation also has a constant k, which equals the wave's velocity squared.

$$\text{(Acceleration of Field)} \quad \partial \boldsymbol{V}/\partial t^2 = k\nabla^2 \boldsymbol{V} \quad \text{(Sharpness of Local Max)}$$

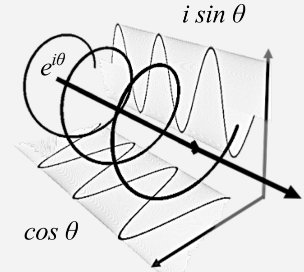
Figure 6-7 Wave Equation

Example 6.4 Imaginary Axis Spiral and Waves

Waves can be represented with the exponential e and rotation θ along the real axis and imaginary $i = \sqrt{-1}$ axis.



The spiral motion of the exponential produces a cosine wave on the real axis and sine wave on the imaginary axis, based on Euler's identity of $e^{i\theta} = \cos\theta + i \sin\theta$.



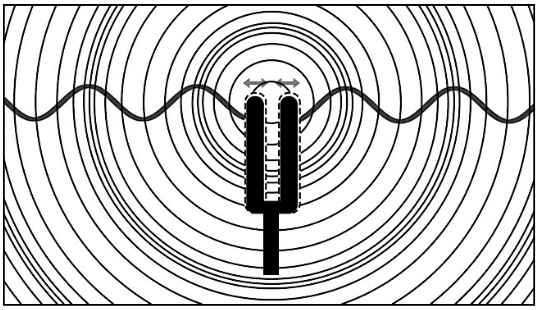


Figure 6-8 Tuning Fork Sound Waves

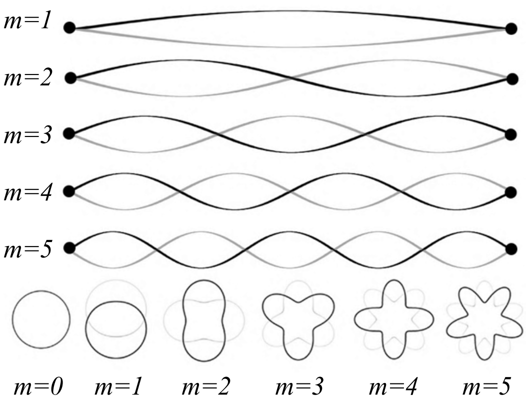


Figure 6-9 Wave Harmonics

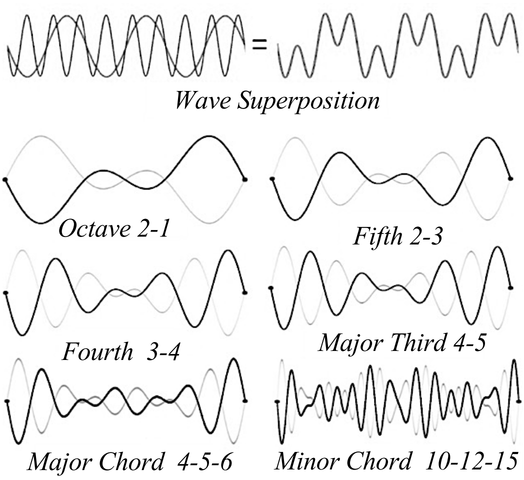


Figure 6-10 Wave Superposition

Sound, which is the oscillation of molecules, can be modeled with waves. After striking a tuning fork, for example, compressed regions of air propagate to create sound. Sound is a longitudinal wave, which means that the wave travels in the direction of motion, unlike a transverse wave, which is perpendicular to motion, likes waves on a string.[101] Sound waves can be efficiently dispersed in resonation devices, like instruments.

Waves can be broken down into different standing wave patterns, called harmonic modes. Figure 6-9 displays the 1st to 5th modes *m* of a linear string and a circular loop. Harmonics only exist in whole number intervals because intervals like 1.5 do not generate standing waves with fixed end points. Multiple harmonics can be superimposed to create waves within a wave. The summation of simple modes creates musical harmonies like octaves, fifths, and chords.

Any complicated waveform can be represented as a summation of harmonic modes, called a Fourier transform. For example, a square wave can be closely approximated with just five smooth waves, as depicted in Figure 6-11. By adding waves ad infinitum, any repeating pattern can be matched. The Fourier transform is an extremely powerful tool in physics and engineering.

Human sensory organs are well adapted to parsing wavelike information from the environment. Hair follicles in the inner ear are sensitive to different isolated sonic frequencies. Similarly, the retina of the eye contains cone receptors that are sensitive to specific frequencies of electromagnetic waves, which humans observe as visible light and colors.

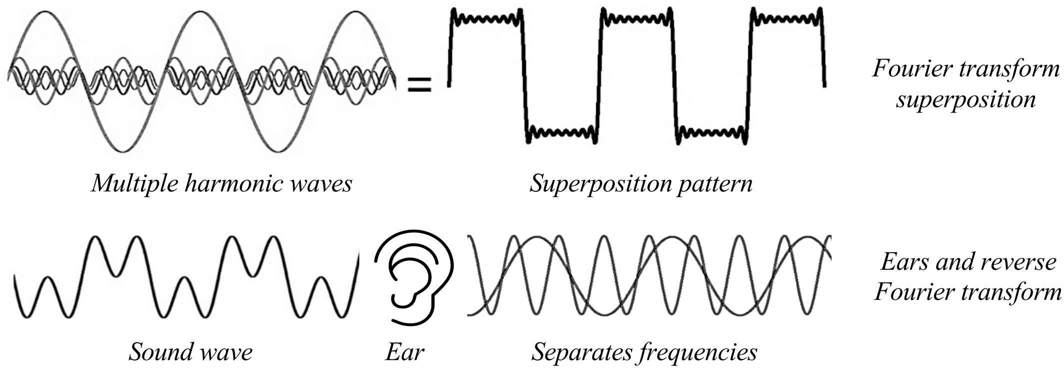


Figure 6-11 Fourier Transform

Simple harmonic waves can describe many systems found in nature. For example, the number of daylight hours available over the course of a year follows a periodic pattern as the Earth makes its revolution around the Sun.[102] This can be graphed as a wave with a period of one year, with a peak at the summer solstice and trough at the winter solstice for locations in the Northern Hemisphere. A wave graph can also represent the cycle of phases of the Moon. These waves are due to the cyclical orbits of the Earth and Moon. The tides in the ocean even follow combinations of periodic waves that have daily, monthly, and other overlapping cycles. Figure 6-12 shows the periodic cycles of the Sun, Moon, and tide data for a month.

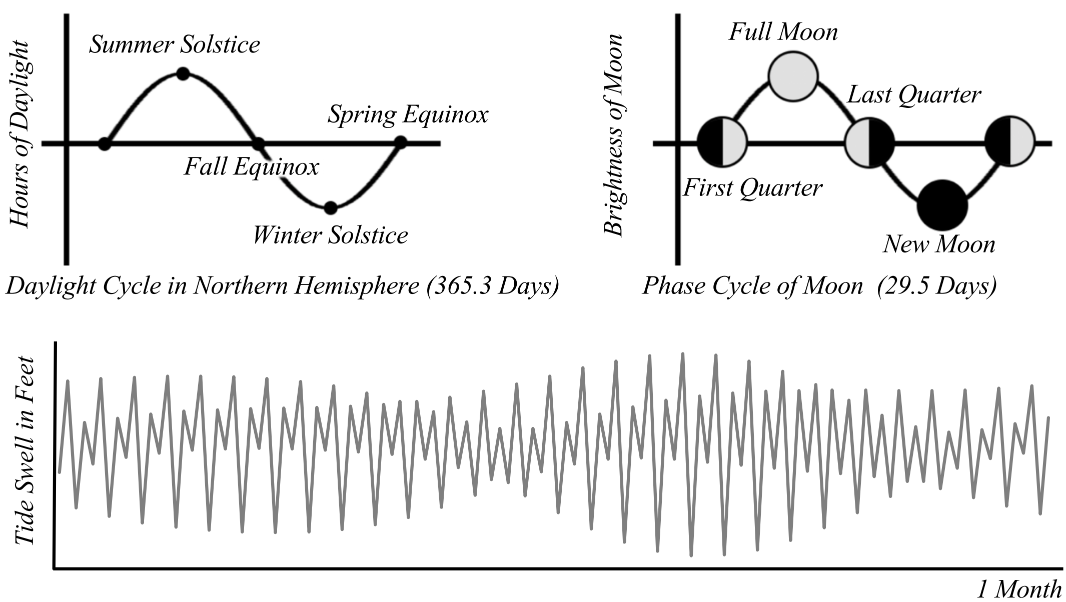


Figure 6-12 Harmonic Cycles of Sun, Moon, and Tides

Surface and Volume Waves

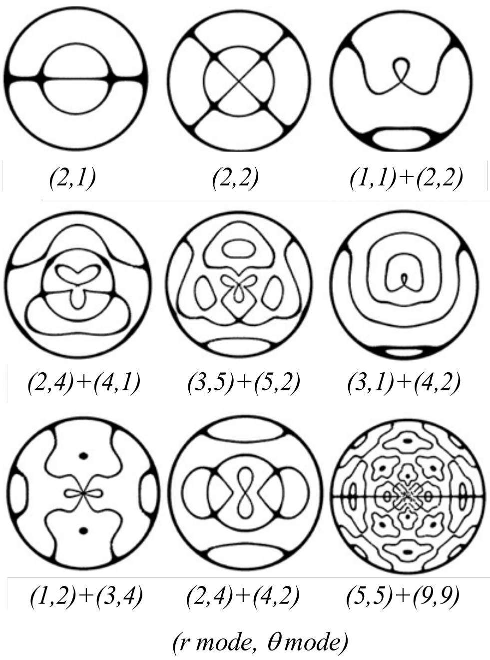


Figure 6-13 Harmonics of a Circular Surface

Solutions for wave harmonics depend on the shapes and dimensions of the system. For example, a circular surface has harmonic modes along the radial axis r and the angular axis θ, written (*r mode, θ mode*) in Figure 6-13. On a square plane, the different modes (*x mode, y mode*) indicate harmonics along the width x and height y. Figure 6-14 represents the different standing waves of a square plane as dark dips and light bumps that oscillate back and forth.

Surface harmonics can be represented with nodal lines, which are the places that remain constant as the wave oscillates. Nodal lines can be physically demonstrated by sprinkling sand on a vibrating surface. Sand tends to avoid vibrating locations and settles into the nodes in a process called cymatics.[103] In addition, surface waves can be superimposed to create more complicated patterns.

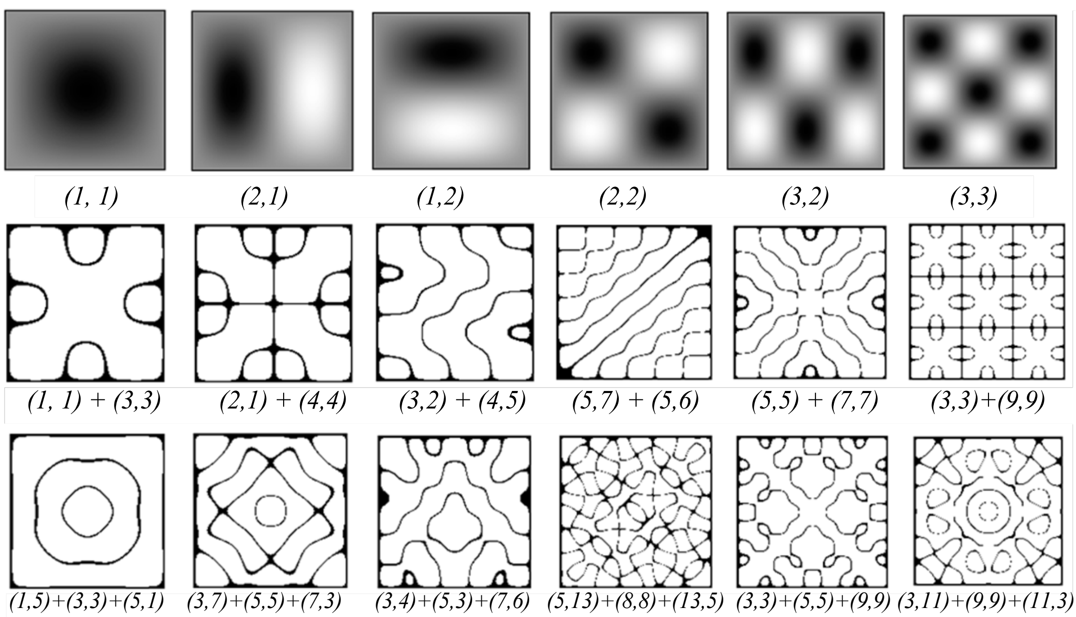


Figure 6-14 Harmonics of a Square Surface (*x mode, y mode*)

Harmonic modes can be extended into three dimensions. Within a cube, the wave modes are defined along the *x, y,* and *z* axes. The darker regions in the cubes are nodal surfaces, where the waves do not oscillate. Sound waves in volumetric spaces can even generate 3-D cymatic patterns.[104] Spherical harmonics are another type of volumetric wave and have applications to modeling electron orbitals and even the cosmic microwave background, which are small energy fluctuations observed across the universe.[105] Volumetric waves can also be superimposed into more complicated combinations.

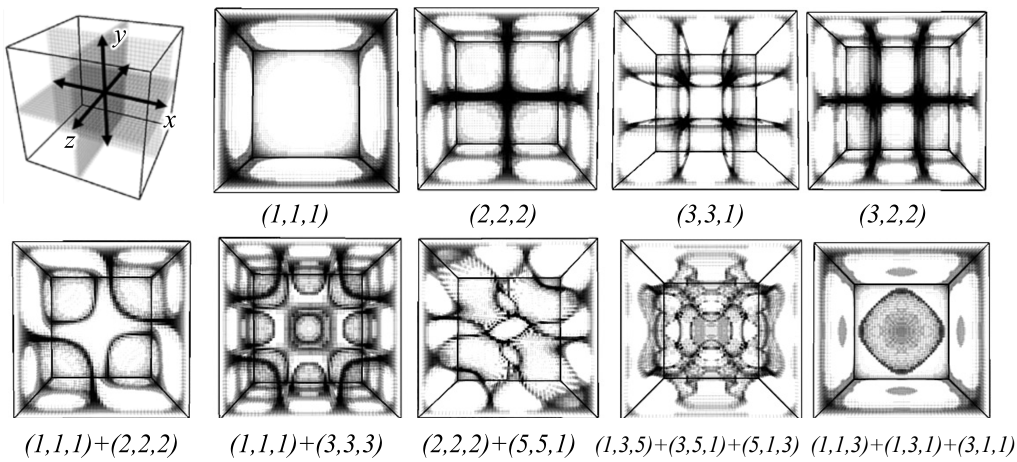


Figure 6-15 Harmonics of a Cubic Volume (*x mode, y mode, z mode*)

Light Spectrum

Waves provide a way to model light, which is an energy transfer process in the electromagnetic field. Light occurs when electric and magnetic fields continuously self-induce one another and travel along a direction of motion. In light waves, the electric and magnetic fields oscillate at perpendicular angles and are synchronized, or in phase. Electromagnetic waves can vary in frequency, wavelength, and energy, but all maintain a constant speed *c* around 3×10^8 m/s.[106]

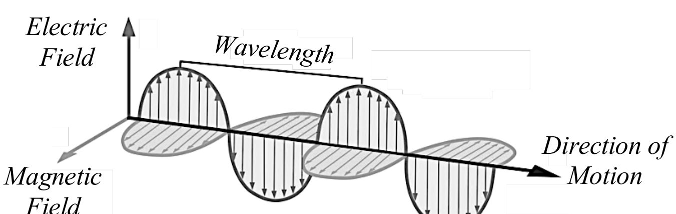


Figure 6-16 Electromagnetic Waves

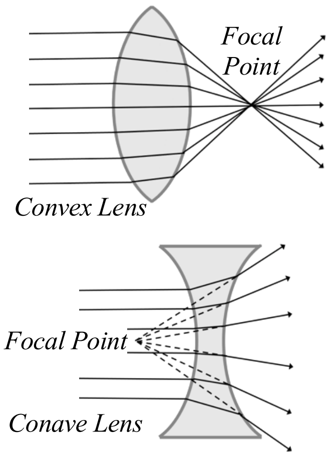


Figure 6-17 Refraction in Lenses

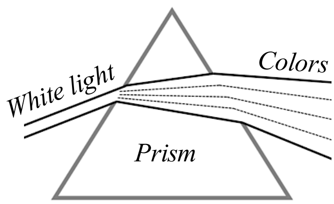


Figure 6-18 Dispersion in Prism

The wave nature of light can be seen in phenomena like refraction and dispersion. As a wave passes through a medium that alters the wave's speed, the angle of motion will change in a process called refraction. Lenses use refraction to direct light toward a focal point and are utilized in telescopes and the human eye to clarify images. Higher frequencies bend at a more acute angle, which is called dispersion. Dispersion is beautifully displayed in a prism which separates white light into different frequencies and colors. Dispersion also enables water droplets to reflect light into a rainbow.[107] While refraction and dispersion are most visible with light, these properties also apply to waves in other mediums, like air or water.

Light exists over a wide spectrum of energy levels and wavelengths. Shorter wavelengths of light have higher energy compared to longer wavelengths. A summary of the electromagnetic spectrum is displayed in Figure 6-19. The wavelength of light also alters the scale of phenomena it will be influenced by. For example, short wavelength gamma rays are diffracted (bend around corners) at atomic scales, while long wavelength radio waves diffract across geologic scales. Visible light, which contains the observable colors, exists at the intramolecular scale, between the size of molecules and biological cells.

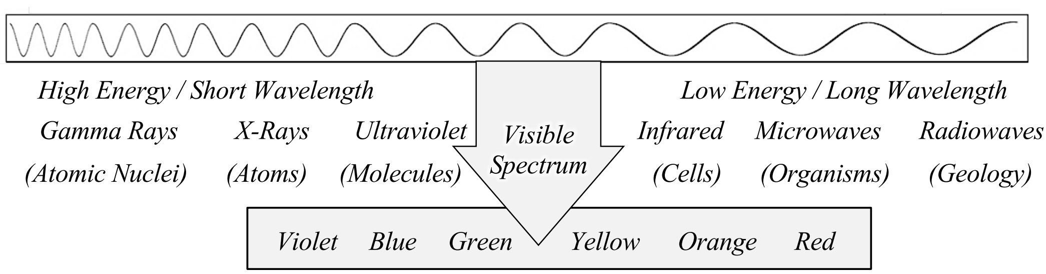


Figure 6-19 Electromagnetic Spectrum

The visible spectrum is the most abundant wavelength that the Sun produces and is critical in biological systems. For example, plants use these wavelengths for photosynthesis and the human eye detects visible wavelengths. Plants and animals also emit a small quantity of photons in and near the visible spectrum, called "biophotons," that are generated via metabolic activity and may be related to cellular communication.[108] Light waves provide a method of transferring energy and information, and they are essential in living systems.

Wave-Particle Duality

Quantum mechanics, introduced in the early 20th century, provided a revolutionary model of nature. An idea within quantum theory is that light has both wave and particle properties, called the wave-particle duality. For example, even though light refracts in lenses like a wave, other properties, such as the photoelectric effect whereby light excites electron energy levels, can only be explained if light moves as discrete packets of energy.[109] In quantum mechanics, discrete units of light, called photons, are packets of energy with both wave and particle properties. The energy E of a photon is proportional to frequency f and Planck's constant h, a commonly used constant in quantum physics.

$$E = h \cdot f$$

Energy of Photon

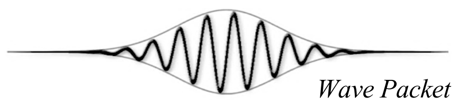


Figure 6-20 Energy Quanta

Wave-particle duality of quantum mechanics is not limited to photons. Louis de Broglie proposed that all physical systems have a wavelength equal to Planck's constant divided by momentum. Planck's constant is extremely small, so only something with very small momentum, like an electron, can have a noticeable wavelength. Yet, following quantum mechanics, all physical systems have a wave component. The wave-particle duality also leads to the uncertainty principle, which asserts that there are unavoidable inaccuracies in measuring the position and momentum of matter due to its wave properties. Quantum physics showed the harsh limitations of a purely particle-based interpretation of nature.

The double slit experiment provided a critical experiment to understand the wave-particle duality. As waves pass through two slits, they create an interference pattern of oscillating peaks. In contrast, when particles diffract through two slits, they create a two-peak distribution, as shown in Figure 6-21. A surprising result of the double slit experiment is that even when one electron at a time is passed through the slits, an interference pattern is still produced after many iterations. This seems impossible because it would require a single electron to interfere with itself and exist in more than one place at a time. This phenomenon is modeled in quantum mechanics through probability fields. The electron's probability wave passes through both slits to interact with itself and create interference. The electron only has a well-defined location, within the bounds of the uncertainty principle, when there is an interaction and measurement event.

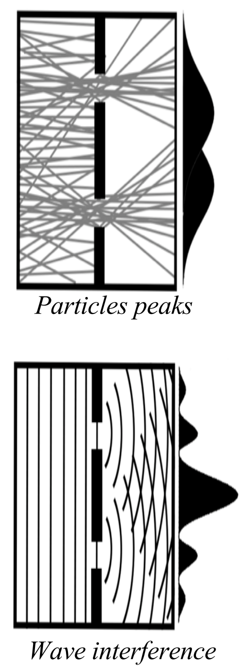


Figure 6-21 Double Slit Experiment

Even more baffling, when instruments are placed to measure which slit the electron passes through, the resulting pattern resembles particles. By measuring the probability field of the electron in one of the slits, the field collapses and does not cause interference. Thus, wave-particles behave like waves when not interacting and like particles when observed via interaction. The surprising results of quantum physics led scientists to dismantle the notion of perfectly defined locations and momentums.

Waves play a crucial role in atomic systems. In an atom, electrons stabilize in a standing wave around the nucleus. The harmonic mode, or orbital level, of the electron also determines its energy. An electron can drop to a lower orbital level by emitting a photon of energy or rise to a higher orbital level by absorbing a photon of energy. In Figure 6-22, the second and third electron energy modes E_2 and E_3 are depicted with an energy transfer via a photon. This figure follows the Bohr model of the hydrogen atom, where negative electrons exist in discrete energy modes around a positive central nucleus.

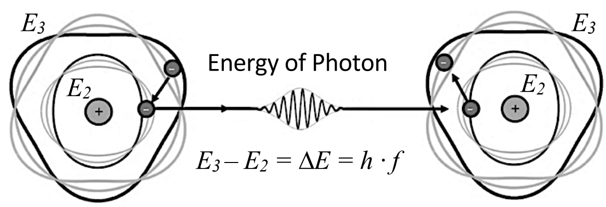
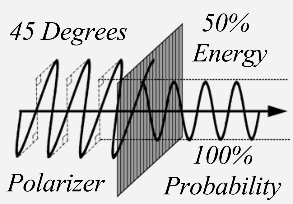


Figure 6-22 Electron Orbital Photon Emission and Absorption

Electron orbitals in atoms and molecules can be more precisely modeled using spherical wave harmonics. These spherical harmonics have different energy levels, or radial modes, as well as angular modes, broken down into the *s*, *p*, *d,* and *f* orbitals. The *s* orbital is spherically symmetric, while the other orbital geometries contain other angular symmetries, as seen in Figure 6-23. Electrons that surround an atom will tend to fall into the lowest energy mode, unless that orbital is pre-occupied with another spin up and spin down electron. The *p*, *d*, and *f* orbitals can also have different orientations on the *x*, *y*, and *z* axes, producing a variety of geometric patterns. The spherical harmonics of electrons provide insights for why particular patterns arise in atomic and molecular systems.

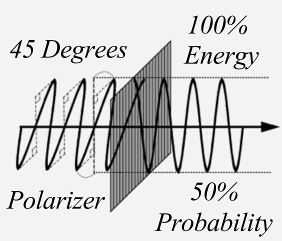
Example 6.5
Quantum vs. Classical Waves

A classical wave passing through a verticle polarizer at 45 degrees will always trasmit half the energy

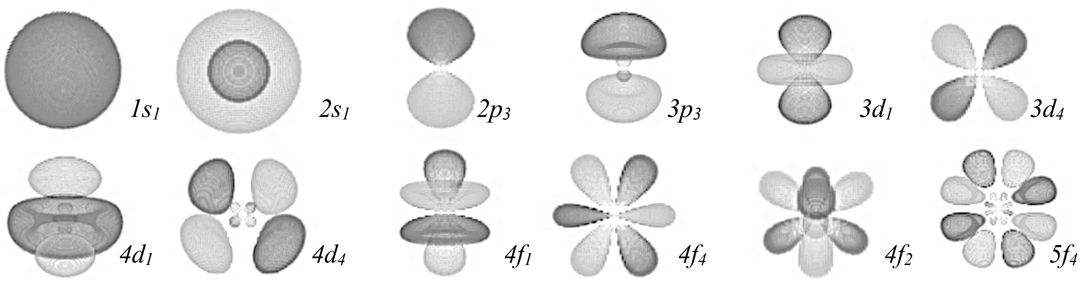


Classical Waves

In contrast, a quantum wave cannot exist in a ½ energy mode. The full energy of the photon will go through the polarizer 50% of the time and be blocked 50% of the time. This creates the same macro effect, but different micro effects. Quantum waves present results that are inherently probabilistic and discrete.



Quantum Waves



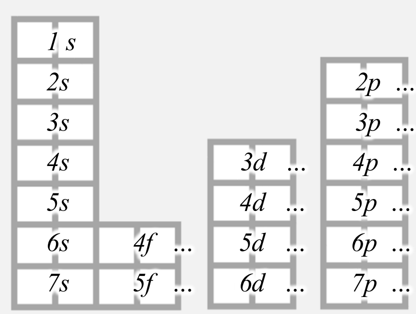
(Energy Level (1-5), Orbitals (s, p, d, f), Number of Orientations of x, y, z axes)

Figure 6-23 Electron Orbital Diagrams

Spherical harmonics play a crucial role in chemistry. The periodic table of elements is organized according to how the *s*, *p*, *d*, and *f* orbitals are filled when electrons balance the charge of the atom. When a given energy level is filled, the atom is less reactive. The so-called "noble gases" in the rightmost column (helium, neon, argon, …) have filled electron energy levels and are the least reactive. In contrast, atoms with partially filled orbital levels tend to bond with other atoms to complete an energy level, driving many kinds of chemical reactions.

Harmonic energy levels even determine the shapes of molecules, such as the tetrahedral geometry of methane. A neutral carbon atom has four electrons in the outer energy level. In a bond with other atoms, such as four hydrogen atoms, the electrons in *s* orbital and *p* orbital create a hybridized sp^3 state. The resulting four sp^3 bonds move away from one another and form a stable molecule with tetrahedral geometry.

Example 6.6 Legend of Periodic Table



s orbital: (2 electrons per level)
p orbital (6 electrons per level)
d orbital (10 electrons per level)
f orbital (14 electrons per level)

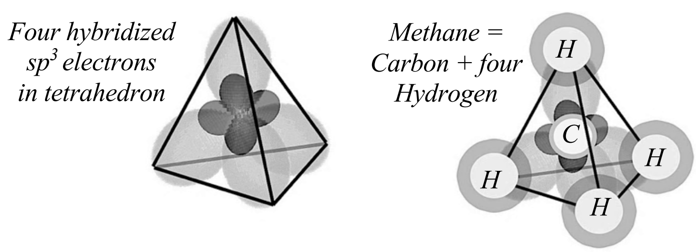


Figure 6-24 Orbital Hybridization and Molecular Bonding

A particularly interesting wave-based phenomenon that can occur in nature is coherence. Coherence occurs when waves are in phase with each other, meaning that the waves are synchronized and have the same frequency. For example, light waves in lasers are all in phase and in coherence. Usually light spreads out, but a laser beam moves in a straight line due to coherence, as shown in Figure 6-25.

Figure 6-25 Coherence in Laser Light

Matter, which also has the properties of waves, can exist in states of coherence that create novel properties. For example, when certain compounds are lowered in temperature, all the electrons can enter a coherent state to create superconductivity, which allows the transfer of electricity with zero resistance.[110] Liquid hydrogen at supercool temperatures can move with zero viscosity.[111] Even though these coherence properties occur from the quantum wave nature of matter, the effects can be observed at the macro-level. For example, all the atoms in a neutron star enter the lowest energy level, called a Bose-Einstein condensate, producing a macroscopic mass sharing a single quantum state.[112]

Nonlinear Flux

Example 6.7
Nonlinear Functions

In a nonlinear function, the function of the whole $f(a + b)$ does not equal the sum of the function applied to the parts $f(a) + f(b)$

Linear:
$f(a) + f(b) = f(a + b)$

Nonlinear:
$f(a) + f(b) \neq f(a + b)$

An important distinction in fluctuating systems is linear rates of change versus nonlinear rates of change. Linear algebraic equations can be graphed as a line, like $f(x) = x + constant$, while nonlinear curves are raised to the second or higher power, such as $f(x) = x^2$. Linear equations follow superposition and add in a linear fashion, meaning $f(a) + f(b) = f(a + b)$. For example, the equation, $f(x) = x$, is only to the first power and follows the law of superposition, such as $f(1) + f(2) = f(3) \rightarrow 1 + 2 = 3$. This linearity enables the function to be easily added together and superimposed. In contrast, nonlinear equations cannot be added as simple summations. The equation $f(x) = x^2$ is nonlinear and doesn't follow superposition as $f(1) + f(2) \neq f(3) \rightarrow 1^2 + 2^2 \neq 3^2$. Nonlinear systems can lead to complexity and unpredictability because the behavior of collections is not equal to a summation of the behavior of the parts.

Nonlinearity can be applied to classify differential equations, which use derivatives to measure rates of change. In linear differential equations, such as gravitation force, the rate of change can be summed following superposition. Even though the gravitational force equation has a squared term ($F \propto 1/x^2$), the gravitational potential energy ($PE \propto 1/x$) at one instant in time can be summed up in a linear fashion.

In contrast, nonlinear rates of change do not easily sum up and create complex results. For example, nonlinear equations are required to model how a wave curls in shallow water, in part due to the fact that waves move at different speeds at the water and air boundary, creating dispersion.[113] Other nonlinear forms of fluids include cloud formations, smoke plumes, and turbulent motion. Nonlinear patterns can be highly sensitive to initial conditions that lead to vastly different large-scale outcomes, known as the butterfly effect. Properties of nonlinear equations include chaos, sensitivity, phase changes, multi-stability, and others in Figure 6-26.

Nonlinear equations can be difficult or impossible to solve. While nonlinear equations can typically be computationally approximated, there is often no analytical method to perfectly predict future states. Most of the physics that describes realistic scenarios, like fluid turbulence or the trajectories of many bodies under gravity, are nonlinear. Nonlinear rates of change create an obstacle to fully predict nature through efficient equations.

Nonlinear rates of change occur in diffusion-reaction systems, which model the diffusion of multiple substances with a balancing reaction on the shared boundary, such as {*more x → more y, more y → less x*}. The normal diffusion equation is linear, and the rate of change is only proportional to one variable. However, two substances diffusing under a boundary reaction can create nonlinearity. From a random start, the substances diffuse into self-organizing shapes, called Turing patterns, that are strikingly similar to fish skin, cat fur spots, zebra stripes, and other living systems.[114] Diffusion-reaction equations show how intricate patterns can develop from simple starting conditions and they may play a role in driving the organization of living systems.

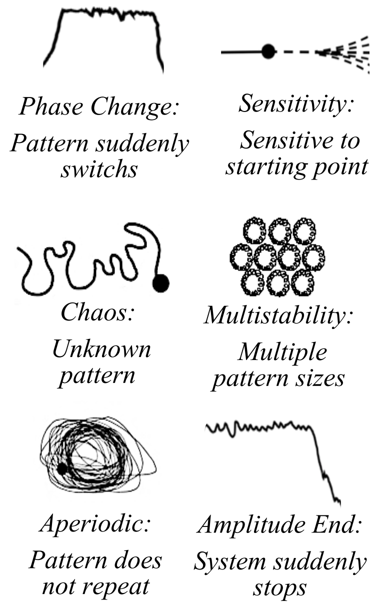


Figure 6-26
Nonlinear Properties

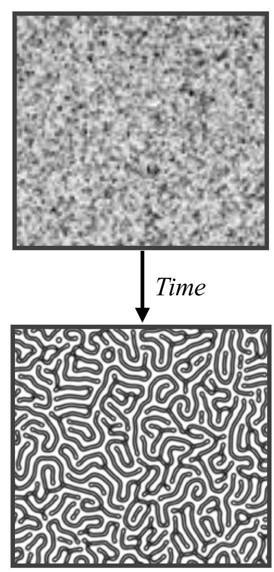


Figure 6-27
Diffusion-Reaction

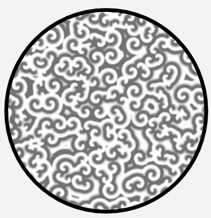
Open Flux Forms

The types of flux considered thus far, such as diffusion and waves, model changing patterns within thermally isolated systems tending towards equilibrium or cyclically repeating. However, particularly interesting patterns of flux occur in systems that are open and continuously exposed to input energy, creating a state of thermodynamic non-equilibrium. Systems exposed to an open source of energy and matter exchange are often nonlinear and reveal a variety of complex patterns.

Convection currents are a common pattern in systems exposed to heat inputs. Convection currents can be seen in a pot of water on a hot stove. Regions of hot water will rise to the top to release heat and regions of cold water will sink to the bottom to gain heat. Convection currents often create toroidal, donut-shaped, cycles as certain regions rise and sink. Convection currents occur in Earth’s atmosphere, oceans, and magma flow driven by the Sun’s heat or Earth’s hot mantle and core, as displayed in Figure 6-28. Earth's convection currents influence weather, wind direction, volcanoes, and tectonic drifts. Convection currents can also form semi-stable circular pockets, called Bénard cells, when dissipating heat. The surface of the Sun has Bénard cells that last an average of about 15 minutes.[115] These convection patterns and cells arise to effectively dissipate heat.

Example 6.8
BZ Reaction

The Belousov-Zhabotinsky (BZ) reaction exists in a state of thermal non-equilibrium. The BZ reaction produces a steady source of chemical energy to drive oscillating spiral-like patterns.



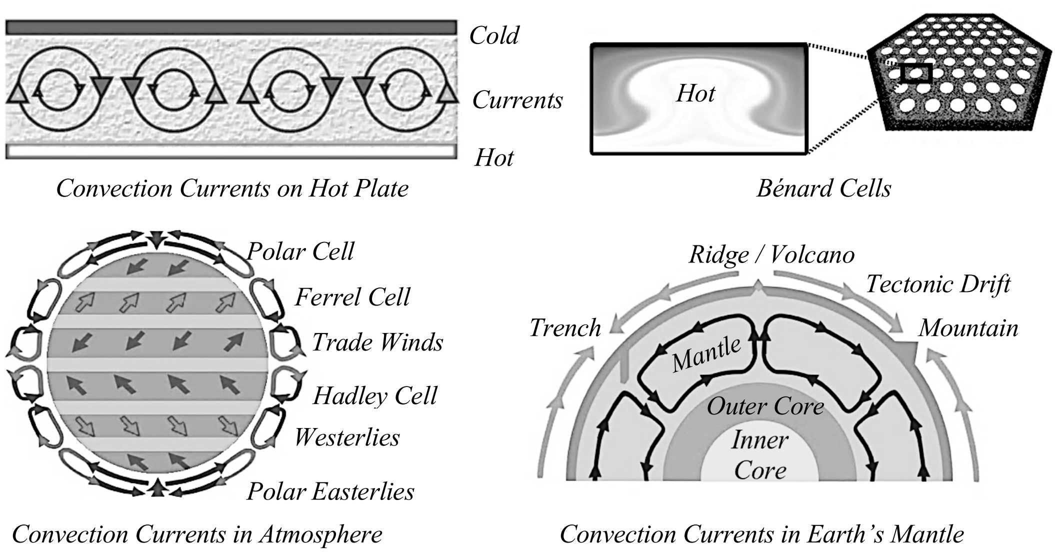


Figure 6-28 Convection Currents

Semi-stable, dissipative, patterns can form in systems exposed to an open flux of matter and energy. For example, a vortex of water can maintain its shape through the continuous motion of fluid and will collapse when the flow is stopped. Semi-stable forms that require an open flux are called dissipative systems, because these forms will dissipate over time without energetic inputs. Tornadoes, which are vortices of air, is another dissipative system that are enabled though a continuous flow of matter and energy in a state of thermodynamic non-equilibrium. Dissipative systems exist between two seemingly opposing elements, as stability is achieved through flux. Dissipating systems are essential to understand structures that maintain a consistent pattern within a flow of energy or materials, including living systems.

Water Vortex *Tornado* *Cyclone*

Figure 6-29 Dissipative Systems

Summary

Flux is a critical concept to understand how natural systems change over time. One important categorization of flux is that some systems of change are linear, like exponential growth, diffusion, and harmonic waves, which can be superimposed in a simple fashion and have predictable solutions. Other patterns of change, like turbulence, convection currents, and dissipative structures, are nonlinear and can be difficult or impossible to perfectly solve, showing chaos and unpredictability. The following chapters will build on this foundation of flux to consider more complex and interconnected systems.

Chapter 7 Symmetry

Example 7.1
Flower Symmetry

Flowers often bloom in a near radial symmetry around a center point to optimize structure and functionality.

Symmetry identifies repeating patterns, providing insights into the underlying structure and relations of systems. Symmetry is defined as a given transformation, noted →, that maps an object *X* back to itself, as shown in Figure 7-1. For example, an image with vertical bilateral symmetry will be transformed to the same initial image when flipped left to right. Symmetrical mappings also express a type of equilibrium, as the change from the initial to final state is zero.

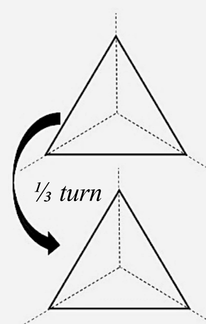
$$S:\{X \rightarrow X\}$$

Figure 7-1 Equation for Symmetry

Many natural systems contain symmetrical patterns, including crystals, plants, and animals. These symmetries are often based on radial changes (rotation of objects) or translational changes (movement of objects over a distance). Symmetry depends on the which transformations are considered, and some symmetries only occur when constraining or expanding sizes, dimensions, as well as considering non-intuitive mappings. Symmetry beautifully displays emergence and provides a higher-level description for how an object repeats, losing the lower-level detail of what the object is. The same symmetry, like bilateral symmetry, can apply to many different objects. Symmetry provides an indispensable tool to understand the structure and relations between elements in a system.

Example 7.2
Triangle Symmetry

An equilateral triangle repeats, and is symmetric, over ⅓ turns. This rotation returns the triangle to the orginal shape.



Circular Symmetries

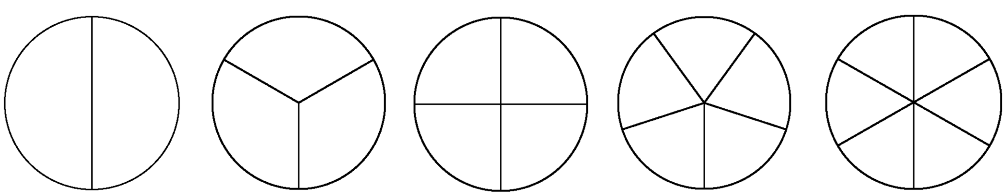
Circular patterns are a common symmetry of systems. A circle is a closed curved path of points on a plane that maintain a constant distance from a center point, which means it is identical as it rotates and symmetrical for any radial turn. Approximate circles can be seen in many natural patterns, such as the silhouette of the Moon or Sun, as well as cross sections of plants, as shown in Figure 7-2.

Circular Outline of Moon *Circular Rings in Onion* *Circular Rings in Beet Slice*

Figure 7-2 Circles in Moon, Onion, and Beet

The sphere is a higher dimensional analog to the circle that extends a fixed radius to a 3-D volume. Both the circle and sphere have important connections to maximally optimal forms. The circle encloses the largest 2-D area through the shortest 1-D curve, and the sphere encloses the largest 3-D volume with the least 2-D surface area. Due to this optimal geometry, approximate circles and spheres often occur in natural systems that follow the principle of least action.

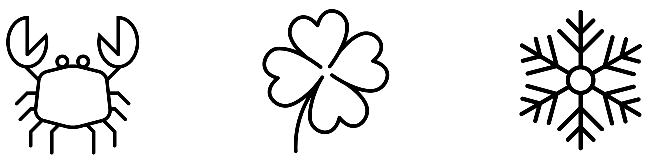
Another category of radial symmetries is to divide rotations into equally spaced symmetries. For example, two-fold symmetry splits a circle into two equal parts, three-fold symmetry splits a circle into three equal parts, and so forth. These fold symmetries correspond to the vertex points of the regular polygons, such as the triangle, square, pentagon, and hexagon. Fold symmetries break a seamless circle into distinct modes that remain identical through specific rotations, like ½ turns, ⅓ turns, ¼ turns, and so on.



Two-fold *Three-fold* *Four-fold* *Five-fold* *Six-fold*

Figure 7-3 Fold Symmetries of a Circle

Symmetrical folds can be observed in many natural systems. For example, animals often have a bilateral symmetry, meaning that there is a left and right reflection along a central axis. This is seen in bilateral vertebrates as well as invertebrates, like crustaceans. Higher circular-fold symmetries can arise in natural systems, like a four leaf clovers, five-pointed starfish, six-fold snowflakes, or the eight-fold juice sacs inside an orange. The chemical base pairs that make DNA helix structures also have fold symmetries when projected to a plane, including 10-fold, 11-fold, and 12-fold symmetries.[116] Symmetrical structures serve as a useful tool to efficiently pack and fill space, as well as interact with the environment in a balanced fashion.



Bilateral Symmetry *Four-Fold Flower* *Six-Fold Snowflake*

Figure 7-4 Fold Symmetry in Nature

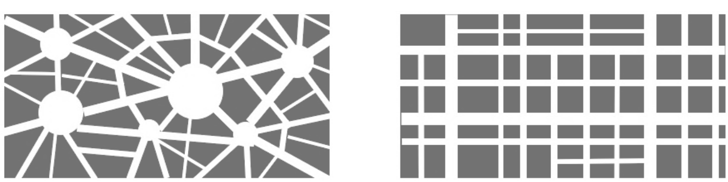
Example 7.3
Matrix Symmetry

Matrices contain a set of values written in rows and columns. A symmetrical matrix is square and remains the same when flipped diagonally, called a transposition.

$$\begin{Bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 5 \\ 3 & 0 & 1 & 6 \\ 4 & 5 & 6 & 1 \end{Bmatrix}$$

Symmetrical axis

Circular and fold symmetries are a common element of architecture and urban design. For example, many cities have plazas which configure surrounding buildings into approximate circles, creating an efficient space for people to gather in a central location. City hubs often tend to expand in concentric circle-like formations around focal points of commercial and residential activity. Bilateral and four-fold symmetry is another common pattern in modern cities. These rectilinear grids create efficient packings of square buildings and enable effective transportation routes for vehicles. These simple fold symmetries are often combined and juxtaposed to create complex designs in architecture and city design.



Circular City Grid *Rectilinear City Grid*

Figure 7-5 Symmetries in City Planning

Packing Circles

Another spatial symmetry occurs when packing circles together around a center point in a circular arc. As shown in Figure 7-6, these circle packings follow the same arrangement as the regular polygons (equilateral triangle, square, pentagon, and hexagon) because each circle radius is constant and corresponds to equal edge lengths. Hexagon packing is unique as it creates just enough space for a seventh interior circle to fit inside the surrounding six with little lost space. Due to this snug fit, the hexagon pattern is the most efficient packing of circles on a plane.[117] Hexagon packings are used in natural systems like honeycombs and carbon nanotubes, which arrange materials in highly space efficient forms.

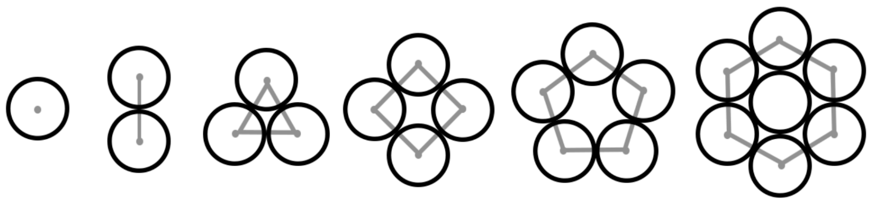


Figure 7-6 Circular Packing and Polygons

Circle packings and their associated polygons can be symmetrically repeated on a plane in a tessellation, or tiling. Periodic tilings have translational symmetry, meaning that patterns repeat when moving along a directional axis. It is possible to make regularly repeating tilings of triangles, squares, and hexagons, as shown in Figure 7-7. Five-sided pentagon tilings present unavoidable empty gaps, but can be arranged with other shapes into tilings, such as aperiodic (non-repeating) tilings corresponding to Penrose tiles.[118] Polygons with seven or more sides also have unavoidable empty gaps but can be combined with other shapes to make space-filling tilings.

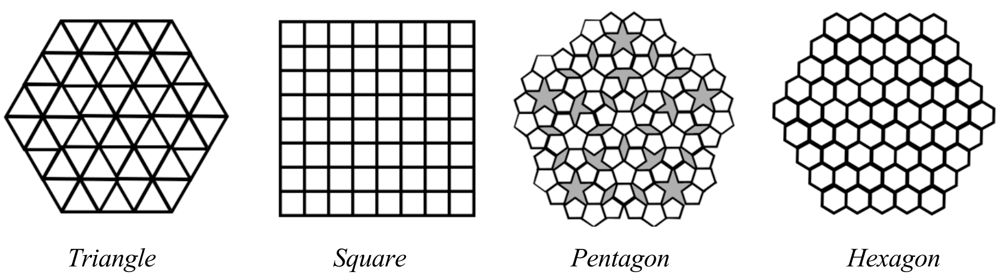


Figure 7-7 Regular and Pentagon Tiling

An interesting pattern utilizing the golden ratio emerges when packing numerous circles as close as possible around a central point. In this pattern, each *n* circle exists at the location of $\sqrt{n}$ radius *(*$\sqrt{1}$*,* $\sqrt{2}$*,* $\sqrt{3}$*, ...)* and at the ϕn angle *(*1ϕ*,* 2ϕ*,* 3ϕ*, ...)*. The golden ratio angle ϕ is defined where two curved length segments follow the golden ratio: $a / b = (a + b) / a$. This angle allows circles to stack in an optimal arrangement with little spacing between the circles. The golden ratio angle naturally occurs in many plants to maximize sunlight collection as leaves grow. Beyond a flat plane, the golden ratio packing (with different radius relations) provides optimal packing arrangements of circles on curved surfaces, like domes and cylinders, and can approximate plant growth on curved surfaces.

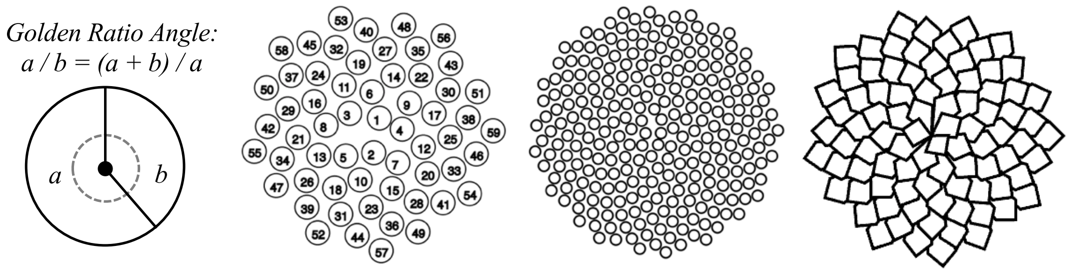


Figure 7-8 Golden Ratio Spiral Packing Pattern

The number of clockwise and anti-clockwise spirals in the golden ratio packing pattern corresponds to the Fibonacci numbers. The Fibonacci sequence is produced by summing the two previous terms for the subsequent term, following *{0, 1, 1, 2, 3, 5, 8, 13, 21, ...}*, and the sequence approaches the golden ratio when comparing adjacent terms. The number of spirals in the golden ratio packing forms follow Fibonacci numbers, like the 21 and 13 anti-clockwise spirals in the sunflower and pinecone on Figure 7-9. Mathematical models provide deeper insight into the mechanism of the golden ratio packing. Interestingly, when modeling plant growth hormones through a set of nonlinear differential equations, the golden ratio packing patterns and Fibonacci numbers emerge as optimal solutions.[119] These shapes increase surface area for photosynthesis, can continually grow, and produce structural stability.

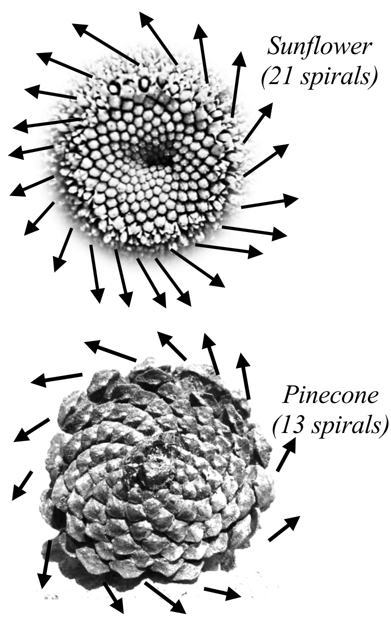


Figure 7-9 Golden Spiral in Plants

Volumetric Symmetry

Common 3-D volumetric forms for modeling systems can be created by extending 2-D forms along a third dimensional axis. For example, a circle extended along a linear axis with a constant radius creates a cylinder, and a circle extended along a linear axis with a shrinking radius creates a cone. Other shapes, like a cube and pyramid, can be generated in a similar fashion by beginning with a square base. A sphere can be formed by rotating a circle on its center axis, and a torus can be formed by rotating a circle along another circular curve. These are a few of the many possible volumetric shapes for identifying spatial patterns in systems.

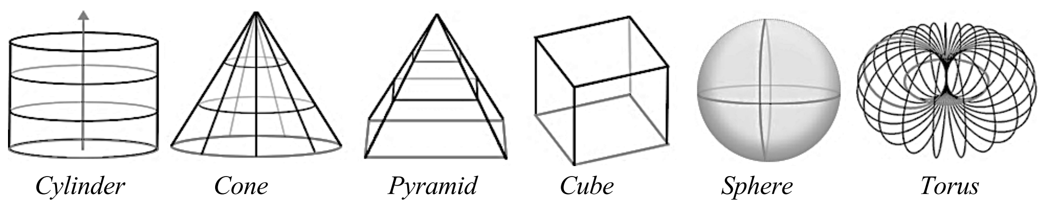


Figure 7-10 Common Volumetric Shapes

Another class of spherical symmetries are the regular polyhedrons. These forms, called the Platonic solids and previously introduced in Figure 4-7, have equal faces, edge lengths, and angles at each vertex. The Platonic solids are common geometric symmetries in volumetric patterns, packing arrangements, and lattice structures. Each of Platonic solid pairs with an inverse dual polyhedron where each point of a smaller solid will fit on the center of the face of a dual solid. The tetrahedron is its own dual, the cube and octahedron are duals, and the dodecahedron and the icosahedron are duals, as shown in Figure 7-11.

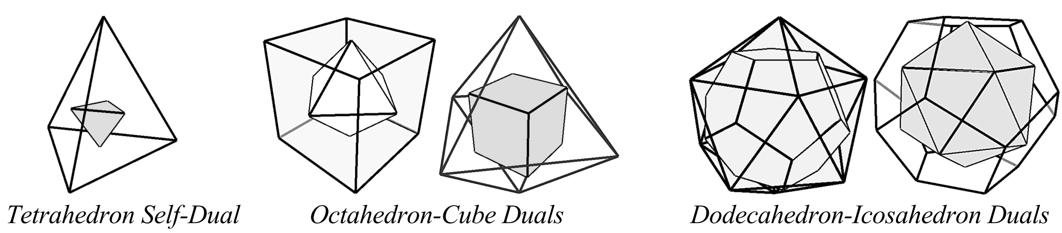


Figure 7-11 Platonic Solids Duals

The Platonic solids present many interrelated patterns. For example, expanding the center of each face on the octahedron will form a star tetrahedron that is inscribed in a cube, as shown in Figure 7-12. The vertices of a dodecahedron and icosahedron can also respectively define five intersecting cubes or octahedrons.

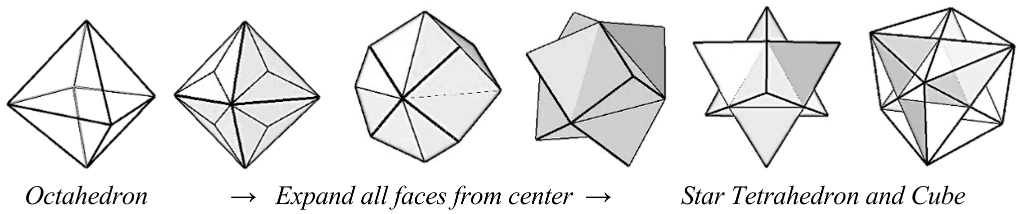


Figure 7-12 Octahedron, Star Tetrahedron, and Cube

Through some clever extrapolations, the Platonic solids map to the points of a 2-D hexagonal lattice. The Flower of Life pattern of hexagonally intersecting circles contains the Fruit of Life, which is a packing of thirteen circles, shown in Figure 7-13. The Metatron's Cube pattern comes from connecting the center of each circle in the Fruit of Life. Metatron's Cube provides an elegant way to show the connection between a 2-D circle lattice and the 3-D Platonic solids. Metatron's Cube aligns with the 2-D projections of the Platonic solids, as drawn in Figure 7-13. It should be noted that the icosahedron and dodecahedron can only be perfectly traced onto the full 3-D Metatron's Cube by looking at the polyhedral in perspective.[120]

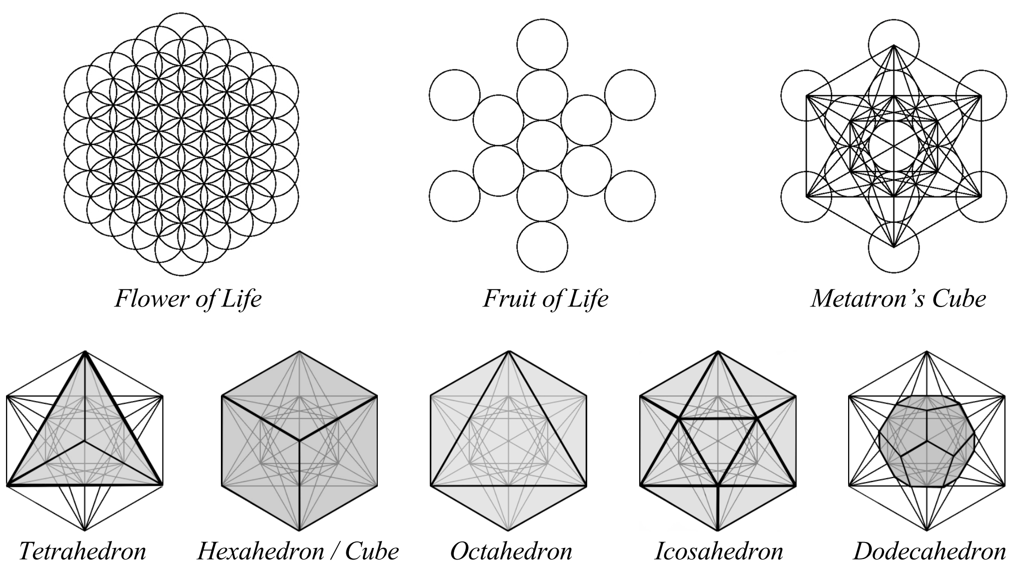
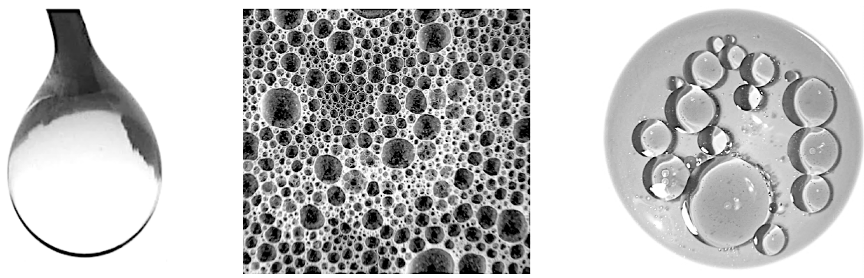


Figure 7-13 Flower of Life, Metatron's Cube, and Platonic Solids

Spherical Packing

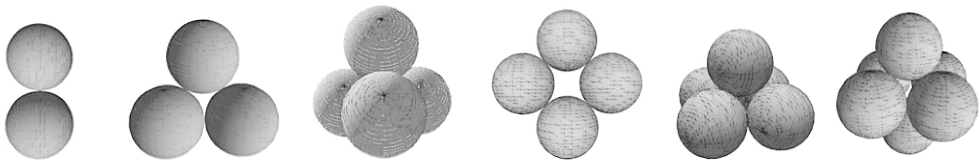
Spherical symmetry often arises in natural systems as the result of forces reaching an equilibrium. A drop of water, for example, will tend to approximate a sphere because this shape contains the least surface area for a given volume and minimizes surface tension. Approximate spherical bubbles can be observed in seafoam or oil drops in water, which form in different sizes to optimally balance the surrounding forces, as shown in Figure 7-14. Planets and stars form in approximate spheres in response to gravitational forces, which tend to configure matter around a center of mass to minimize potential energy. From a physical lens, spherical shapes arise as optimal solutions to balance forces, reduce action, and minimize surface area.



Water Droplet *Air Bubbles in Sea Foam* *Circular Oil Droplets*

Figure 7-14 Spherical Drops and Bubbles

Much like circular packing in two dimensions, various symmetrical forms arise as spheres are packed closely together around a center point. When two spheres are packed together, they will meet along a line. Three spheres will pack in a triangular arrangement and four spheres will create a tetrahedral arrangement. Five spheres produce a two-pointed tetrahedron. Going further, six spheres will pack optimally in an octahedral arrangement, where cross sections form a square shape, shown in Figure 7-15. Even higher numbers of spheres packed to a center point will create a wide variety of symmetric and asymmetric forms. These optimal spherical packings can be physically demonstrated by adjoining approximately spherical balloons together at a center point. The balloons will be pulled to a center point but have a repelling force when too close to a neighbor. In finding an equilibrium between pulling and pushing forces, the balloons tend to follow the same forms as optimal spherical packings.

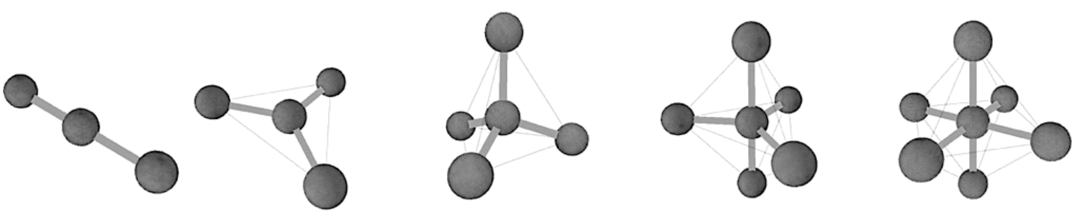


Tetrahedral Packing of Four Spheres *Octahedral Packing of Six Spheres*

Two Balloons (Linear) *Three Balloons (Trigonal)* *Four Balloons (Tetrahedron)* *Six Balloons (Octahedron)*

Figure 7-15 Spherical Packings

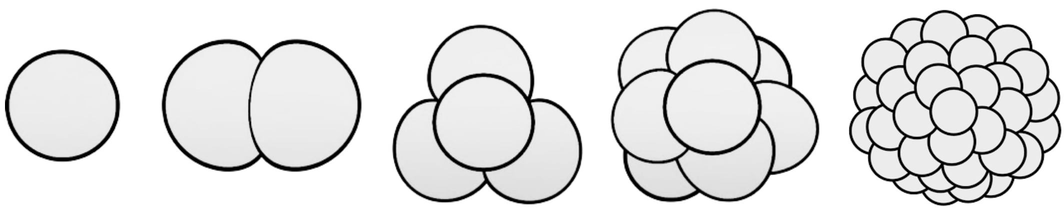
Optimal spherical packing symmetries play a critical role in chemical structures. Due to repelling electric forces, electrons orbiting atoms and electron bonds connecting atoms will tend to move away from one another. Electron bonds connecting two atoms to a central atom will tend to present in a linear shape to be the furthest away from one another. Following the same pattern as the optimal packing of spheres, higher numbers of electron pairs will form according to the triangle, tetrahedron, and octahedron. Seven, eight, and nine electron pairs go on to produce different geometric patterns. Additionally, if there are lone pairs of electrons that are not bonded to any atom, it can alter the molecular geometry. For example, the water molecule H_2O has three electron pairs, but only bonds with two hydrogen atoms, resulting in a bent alteration of the linear geometry.



Linear 2 Pairs	*Trigonal* 3 Pairs	*Tetrahedron* 4 Pairs	*Trigonal Bipyramid* 5 Pairs	*Octahedron* 6 Pairs
(CO_2) *Carbon Dioxide*	(BF_3) *Boron Trifluoride*	(CH_4) *Methane*	(PF_5) *Phosphorus Pentafluoride*	(SF_6) *Sulfur Hexafluoride*

Figure 7-16 Molecular Geometry of Electron Pairs

Spherical packing arises in animal morphology patterns, starting from the earliest stages of life. The first egg cell of an embryo is approximately a sphere. This initial egg cell then splits through cell division to become two cells with linear symmetry. These two cells divide once more into four cells, which often forms into a tetrahedral arrangement, and these four cells split into eight and so on, displayed in Figure 7-17. Some embryos form in a square at the 8-cell stage, but tetrahedron arrangements have shown to be more viable to survive.[121] From there, the embryo cells grow through many asymmetrical arrangements and eventually create an approximate spherical shell called a blastocyst, which differentiates inner versus outer cells. As the blastocyst develops, the inner layer forms the digestive tract, the middle layer forms into muscles, and the outer layers creates skin.



1 Cell *2 Cells: Linear* *4 Cells: Tetrahedron* *8 Cells: Skewed Cube* *Blastocyst*

Figure 7-17 Geometry of Early Embryo

Plant morphology is another domain where spherical packing forms occur. For example, seed pods can have approximate three-fold, four-fold, five-fold, and even dodecahedral symmetry, as shown in Figure 7-18. Fruit and flowers also approximate optimal packing when growing. These packing forms have benefits for plants like efficiently filling space, increasing structural integrity, and gaining more exposure to sunlight.

3-, 4- , 5-Fold Seeds *Dodecahedron Seed* *Packing in Berries* *Packing in Flowers*

Figure 7-18 Packing Symmetries in Seed Pods, Fruits, and Flowers

Microscopic biological systems even utilize icosahedral symmetries. For example, viruses take advantage of icosahedral symmetry, along with helical and spherical patterns, to protect their RNA material in an efficient manner.[122] Also, single-cell organisms called radiolarians can create a wide range of intricate mineral skeletons in spherically symmetrical distributions to protect the outer regions of the cell, as shown in Figure 7-19. These are some of the many spherically symmetrical structures in biology.

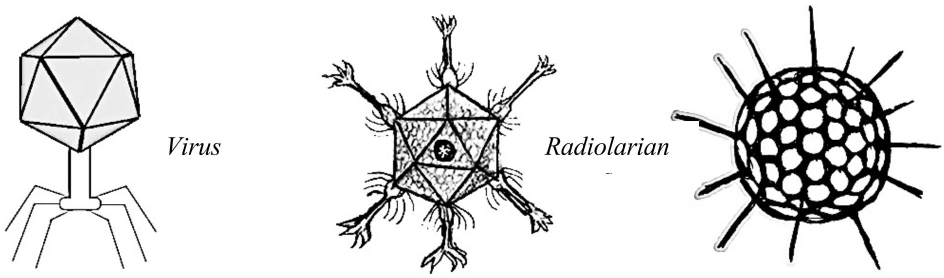
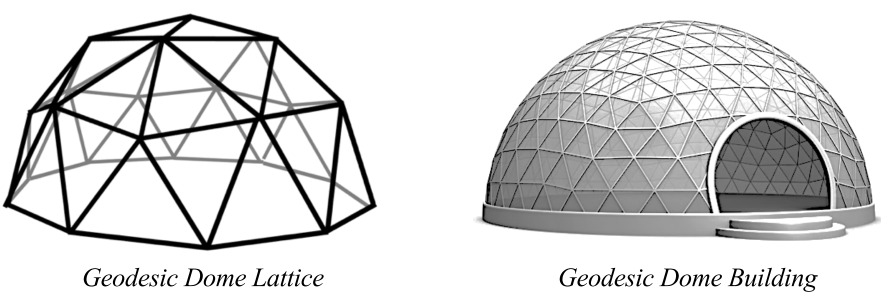


Figure 7-19 Icosahedral Symmetry in Viruses and Radiolarians

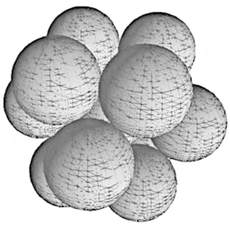
Spherical symmetry can be applied in architecture for efficient designs. Buckminster Fuller was an innovative designer who often utilized spherical geodesic domes for their high levels of structural integrity. These structures efficiently enclose large volumes with a low amount of material. Geodesic domes can reduce construction time compared to conventional architecture, because each side is the same length and many angles repeat. Geodesic domes were one of the many innovations Fuller envisioned as part of the "design science revolution," which is a new way to design solutions to global problems by thinking in terms of synergistic patterns and interrelated systems. The geodesic dome is just one of many techniques that can save resources and accomplish system-wide benefits.



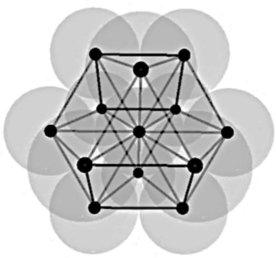
Geodesic Dome Lattice *Geodesic Dome Building*

Figure 7-20 Geodesic Domes

Lattice Structures



Cuboctahedron



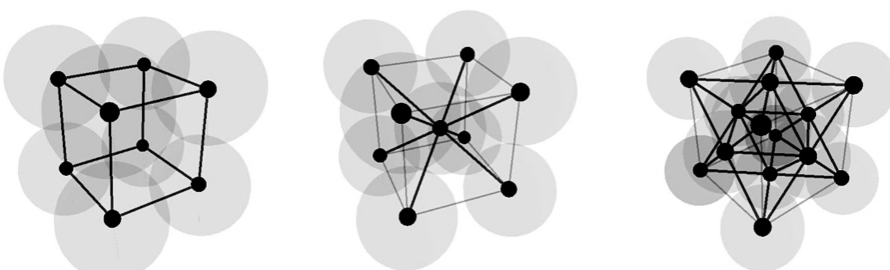
Cuboctahedron Lattice

Figure 7-21 Cubic Closed Packing

Repeating volumetric patterns, called lattices, are an important class of symmetry in systems. A highly optimal lattice is the cubic closed packing, which consists of points on each of the vertexes and the middle of each face of a cube. Each sphere in this lattice is the same distance away from its nearest neighbors and the overall shape is extremely strong and dense. It is believed that closed cubic packing is the densest possible way to pack equally sized spheres in 3-D, which is known as the Kepler Conjecture.[123]

One formulation of this dense packing is to have twelve spheres surrounding a center sphere in a cuboctahedron geometry, as depicted in Figure 7-21. The cuboctahedron can be formed by truncating (replacing end points with a plane) to either a cube or octahedron. Cuboctahedrons can then be iteratively stacked in a lattice to fill a volume. Cubic closed packing can also be constructed through stacking tetrahedrons and octahedrons, as each cuboctahedron consists of eight tetrahedrons and six half-octahedrons. Projecting the cubic closed lattice onto a plane also maps to a triangular lattice, providing a connection between optimal 3-D sphere packing and optimal 2-D circle packing.

Lattice symmetries are useful tools to model the periodic arrangement of atoms in crystals. Crystalline structures can be approximately modeled through lattices with spheres of equal or various sizes. Most crystals are based on either the cubic, body-centric cubic, or cubic closed packing as shown in Figure 7-22.[124] These cubic-based lattices can also be skewed and bent, which result in seven classes of crystal geometry.[125]



Cubic *Body-Centered Cubic* *Cubic Closed Packing*

Figure 7-22 Cubic Crystal Orientations

The internal structure of metals often displays cubic closed packing and hexagonal closed packing symmetry, which is a different form of cubic closed packing that alternates every layer (*A, B, ...*), instead of every third layer (*A, B, C, ...*), as shown in Figure 7-23. At room temperature, the atoms of aluminum and gold tend to align in cubic closed packing, while zinc and titanium tend toward hexagonal closed packing.[126] In metals, electrons are not bound to any given atom and pervade the entire substance, which produces properties such as electrical conductivity and malleability. Metal lattices often deviate from a perfect lattice ordering through impurities, being bent into deformed shapes, or when mixed with other metals in alloys.

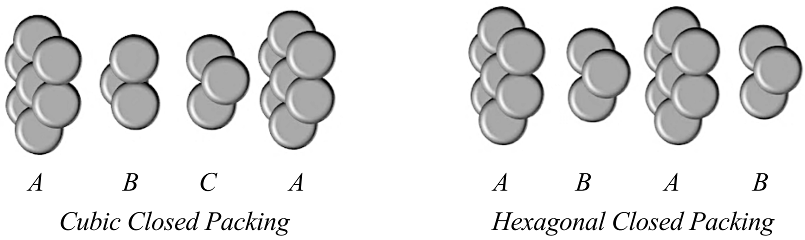


Figure 7-23 Cubic Closed Packing Vs. Hexagon Closed Packing

The atomic lattices in crystal formations beautifully display symmetry. Salt is a common example of a crystal and is composed of positively charged sodium atoms and negatively charged chloride atoms. The attraction of charged atoms create ionic bonding and form a closed cubic lattice, shown in Figure 7-24. Diamonds crystals are composed of carbon atoms. Each carbon atom has four covalent bonds, caused by sharing electron pairs, which forms a stable diamond-cubic lattice with tetrahedron junctures. Ice forms another type of crystal where hydrogen and oxygen atoms stack in layered hexagonal rings. The lattices structures found in crystals influence properties such as transparency, hardness, and electrical behaviors.

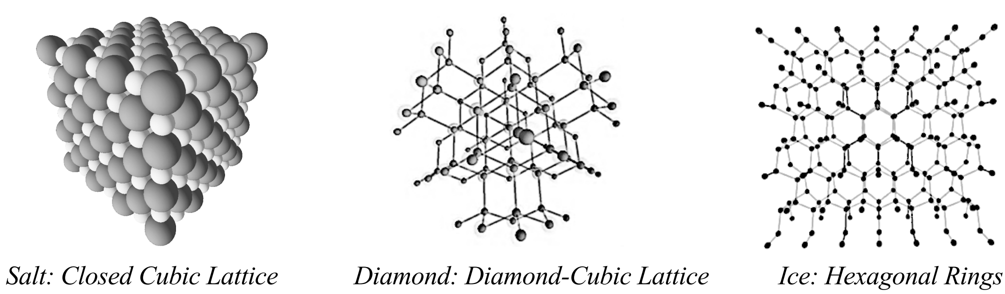


Figure 7-24 Crystal Lattices in Common Materials

Calcite

Fluorite

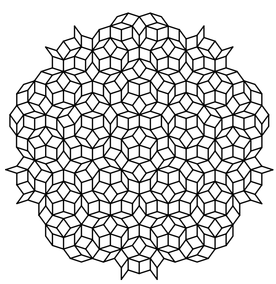
Pyrite

Figure 7-25 Macroscopic Crystal Forms

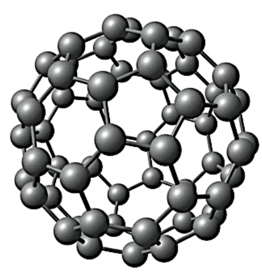
In realistic conditions, many factors can cause crystals to deviate from a perfectly uniform lattice. Material impurities, temperature, pressure, and other environmental factors can alter the shape of a crystal lattice. Additionally, atomic nuclei and electrons are continuously vibrating from temperature and the wave-particle duality. Lattices represent the average geometry among energetic fluctuations and are not perfectly defined at a given time.

While not perfectly determined, the macro-level geometric forms of crystals are influenced by microscopic lattice symmetries. For example, quartz crystals often terminate, or end, on a six-sided point due to the hexagonal symmetry of the atomic lattice. Fluorite crystals have a body-centered lattice, resulting in the approximate octahedral shape, as shown in Figure 7-25. Pyrite metal forms approximate cubes and dodecahedrons from its cubic lattice. These are a few examples that show how the underlying atomic lattice symmetry has an influence on the overall crystal shape.

Crystal lattice symmetries are almost exclusively based on the tetrahedron, octahedron, and cube, rather than the icosahedron or dodecahedron. It was believed to be impossible for icosahedral crystal symmetry to occur because there are no repeatable lattice patterns of icosahedrons or dodecahedrons to fill a volume without empty gaps. Contrary to common thought, Dan Shechtman discovered in 1982 that some crystals, called quasicrystals, have an aperiodic icosahedral symmetry, a discovery for which he was awarded the Nobel prize.[127] In quasicrystals, atoms form into groups, such as the Bergman cluster, which creates a non-repeating aperiodic lattice.[128] When shining a light on a quasicrystal, the resulting diffraction patterns have five-fold symmetry similar to aperiodic Penrose tilings.[129] Another example of icosahedral symmetry is buckminsterfullerene, or “buckyball”, nanoparticles formed out of sixty carbon atoms in the shape of a truncated icosahedron, as depicted in Figure 7-26.[130]



Aperiodic Penrose Tiling



Buckyball of 60 Carbon Atoms

Figure 7-26 Quasicrystals and Icosahedral Symmetry

Higher Dimensions

Spatial symmetry can be extrapolated to higher dimensions, such as 4-D space, which generates various non-intuitive properties. A common element in 4-D space is the hypersphere, which is a higher dimension analog to a 3-D sphere. While a curved surface is sufficient to define the boundary of a sphere, a curved volume is required to define the boundary of a hypersphere. In a hypersphere, it is possible to continuously move in a straight line and never reach the edge of the warped volume, which defies common notions of spatial relations. A hypersphere can also be mapped to a sphere where each point is extended to a circle, or fiber bundle, that intertwines through a process called a Hopf fibration. These models warp a flat Euclidean perspective of spatial systems.

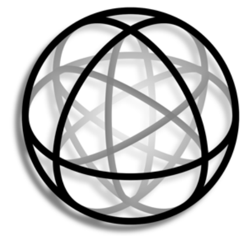


Figure 7-27 Hypersphere

The 3-D regular polyhedron have higher dimensional analogs, called 4-D polytopes. While the 3-D Platonic solids are made by stitching flat polygons together, the 4-D regular polytopes are made by stitching together multiple Platonic solids. The hypercube, for example, is constructed with eight cubes that warp around one another. In total, there are six regular 4-D polytopes, shown in Figure 7-28. Five of these polytopes are higher dimension analogs to the Platonic solids and the sixth is called the 24-cell. In 5-D and higher, the number of regular polytopes drops to three, thus 4-D has the most regular polytopes.

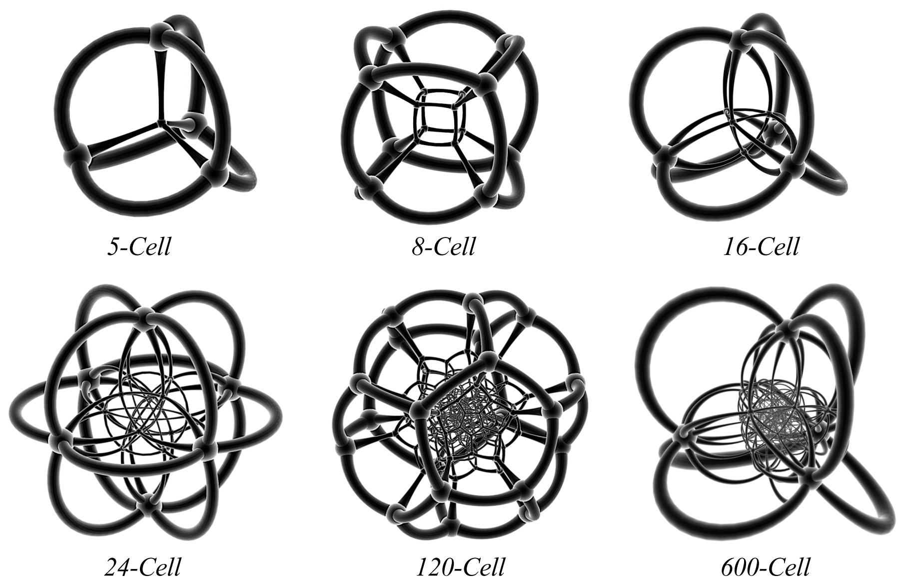


Figure 7-28 Four-Dimensional Regular Polytopes

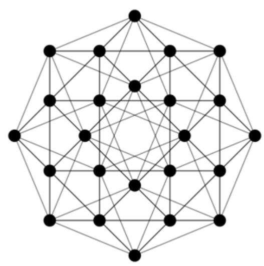
The 4-D polytopes have unique properties beyond 3-D forms. The 24-cell is particularly interesting as it is its own self-dual and has no direct analog in 3-D. The 24-cell lattice is the optimal 4-D hypersphere packing, like the closed cubic lattice is in 3-D and the triangular lattice is in 2-D. The 120-cell is also an interesting 4-D polytope, which may be useful in physics applications. For example, the 120-cell may be helpful in modeling the shape of the universe and the cosmic microwave background.[131]

Example 7.4 Densest Hypersphere Lattices

The optimal lattice packings of the circle, sphere, hypersphere, and higher dimensional spheres are known up to 8-D, as well as in 24-D. These packings are below:

2-D: Hexagon lattice
3-D: Closed cubic packing
4-D: 24-cell packing
5-D: Closed cubic analog
6-D: Cross section of E_8
7-D: Cross section of E_8
8-D: E_8 Lattice
24-D: Leech lattice

The E_8 term references an exceptional symmetry group that arises when studying continuous spaces, called Lie groups. The Leech lattice is highly symmetrical and allows for a very large number of hyperspheres to touch a common unit in 24-D.



- *24-Cell is a 4-D regular solid made with 24 octahedrons*
- *There are 24 vertex points with the locations:*
  (±1, ±1, 0, 0) , (0, ±1, ±1, 0)
  (±1, 0, ±1, 0) , (0, ±1, 0, ±1)
  (±1, 0, 0, ±1) , (0, 0, ±1, ±1)

Figure 7-29 24-Cell Orthogonal Projection

Even higher dimensions can be used to analyze natural systems. In crystallography, some diffraction patterns of icosahedral quasicrystals can be modeled with 5-D lattices. Another example of higher dimensional models is the Kaluza–Klein theory that extends general relativity into 5-D spacetime, where a line translates to a small cylinder. This model was proposed to account for quantum spin within gravity. The 6-D Calabi-Yau manifold, as depicted in Fig 7-30, is another model used to describe bundled up dimensions in string theory. String theory attempts to unify general relativity and quantum mechanics. In string theory, higher dimensions, up to 10-D or 11-D, are bundled up at the quantum level.

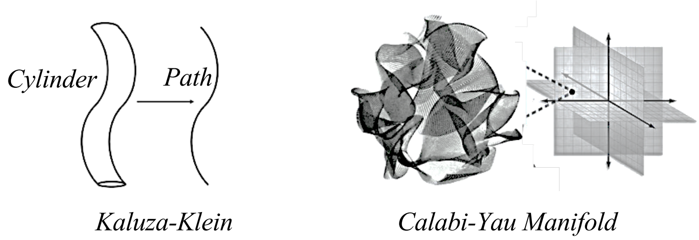


Figure 7-30 Higher Dimensional Physics

Symmetry Groups

Groups provide a powerful and generally way to study symmetry. A group is the set of all symmetries that apply to an object. For example, consider an equilateral triangle where each vertex is indistinguishable, but is labeled *a, b,* and *c* to track changes. The identity symmetry includes transformations that leave the triangle as it originally is, like a 360° rotation. Rotating the triangle 120° to the right is another symmetrical operation, but it switches the vertices into a new clockwise order (*c*, *a*, *b*). The triangle can also be rotated -120°, or 240°, for a new symmetry with vertex order (*b*, *c*, *a*). Additionally, the triangle can be flipped along any of the *a, b,* or *c* axes to create new orientations of the vertices. These six transformations comprise the symmetry group of the triangle and represent every possible mapping the preserves the structure.

Example 7.5 D_n Group

The dihedral group D_n includes symmetries of regular n-sided polygons.

D_1 = Line (1 turn)
D_2 = Rectangle (½ turn)
D_3 = Triangle (⅓ turn)
D_4 = Square (¼ turn)
D_5 = Pentagon (⅕ turn)
D_6 = Hexagon (⅙ turn)
....

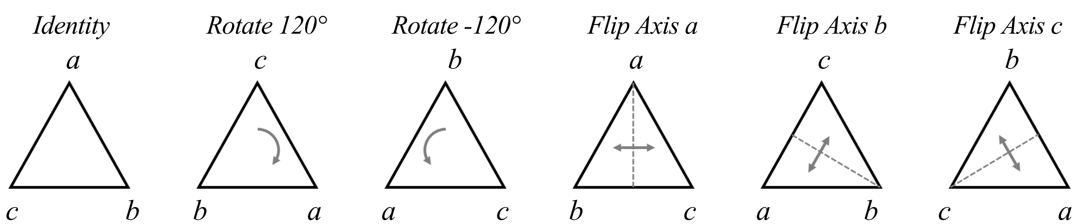


Figure 7-31 Symmetry Group of an Equilateral Triangle

Groups can be used to classify shapes, like the uniform polytopes. The Coxeter group includes various families of polytopes (A_n, B_n, ...) and denotes the dimension in the subscript. The A_n Coxeter group includes polytopes based on triangles, like the 2-D triangle (A_2), 3-D tetrahedron (A_3), 4-D 5-cell (A_4), as well as higher dimensional analogs. The B_n group members are based on squares, like the square (B_2), cube and octahedron (B_3), hypercube (B_4), and so on. The H_n group contains the pentagon (H_2), dodecahedron and icosahedron (H_3), and the 120-cell (H_4), but there are no pentagonal polytopes in 5-D or higher. Additionally, there are five exception cases that create irreducible symmetries, which are the hexagon (G_2), 24-cell (F_4), and three other exceptional groups E_6, E_7, and E_8 that create uniform polytopes (through not perfectly regular) with new symmetries in 6-D, 7-D, and 8-D, called 1_{22}, 2_{21}, and others. These polytope families are summarized in Figure 7-32.

A_n: Triangle, Tetrahedron, 5-Cell, 5-simplex, 6-simplex ...

B_n: Square, Cube, Octahedron, 8-cell, 16-Cell, 5-cube, ...

H_n: Pentagon, Dodecahedron, Icosahedron, 120-Cell, 600-Cell

G_4: Hexagon

F_4: 24-Cell

E_6: (1_{22}, 2_{21})

E_7: (1_{32}, 2_{31}, 3_{21})

E_8: (1_{42}, 2_{41}, 4_{21})

Figure 7-32 Coxeter Groups and Polytope Families

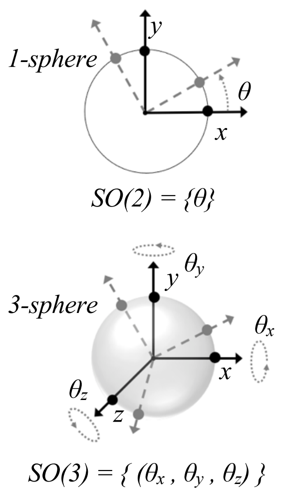


Figure 7-33
Rotation Groups

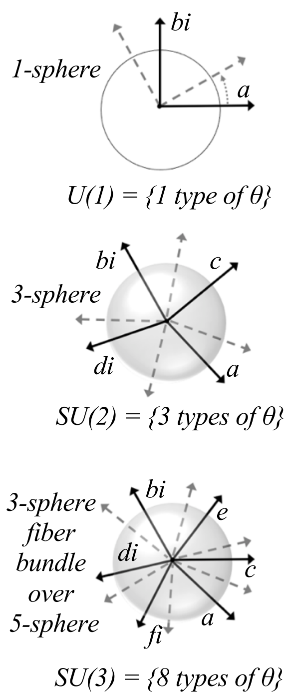


Figure 7-34
Unitary Groups

A widely used group, with deep physical applications, are the special orthogonal groups *SO(n),* which are symmetrical across all possible rotations in *n*-dimension space. The special orthogonal group *SO(2)* includes all the continuous rotations of a 2-D plane. *SO(2)* can be geometrically mapped to the path of the circle (or 1-sphere) that remains symmetrical under continuous rotations of the single angle θ. The *SO(3)* group is the set of 3-D rotations, which includes three unique rotations on each axis θ_x, θ_y, and θ_z. *SO(3)* can also be mapped to the 3-D volumetric surface of a hypersphere (3-sphere). Figure 7-33 shows *SO(2)* and *SO(3)* with examples of rotated coordinates in dotted lines. These continuous rotational symmetries are very general and contain polytope symmetries as subgroups with finite rotations.

The size of rotation groups refers to the dimensional size of possible transformations, which is distinct from the dimensions of the spaces being rotated. For example, even though *SO(2)* is rotating a 2-D plane, it has a size of 1-D because there is just one continuous rotation variable θ that comprises the path of the circle. *SO(3)* rotates 3-D spaces and also has a size of 3-D, with three distinct possible rotations on each axis θ_x, θ_y, and θ_z. More generally, the dimensionality of *SO(n)* rotations follows $\frac{1}{2}n(n\text{-}1)$.

Another foundational rotation group useful for physics is the Unitary Group U_n, whose members are the set of possible rotations of *n*-dimensional complex numbers. Complex numbers have both real and imaginary components ($a + bi$), where $i^2 = -1$. The Unitary group *U(1)* describes the rotations of one complex number, which has both a real axis and an imaginary axis. *U(1)* can also be mapped to *SO(2)*, because both rotate along two axes and can be geometrically transformed into the circle.

The Special Unitary groups, *SU(n),* are called "special" because these groups only include new rotations not in *U(1)*. The *SU(2)* group considers the rotational symmetries of two complex numbers, written ($a + bi$) and ($c + di$), and contains three new types of rotations. *SU(2)* and *SO(3)* are very similar because they both have three distinct rotation variables, which can be mapped to a 3-sphere. *SU(3)* rotates three complex numbers, ($a + bi$), ($c + di$), and ($e + fi$), and has a size of 8-D with 8 distinct rotations. In general, the dimension of *SU(n)* follows the relation of $n^2\text{-}1$. *SU(3)* also has an interesting geometric mapping, which equals a Hopf fibration of 3-sphere fiber bundles over a 5-sphere manifold. These unitary groups are summarized in Figure 7-34.

The unitary groups *U(1), SU(2),* and *SU(3)* are essential to understanding the forces in the Standard Model of particle physics. The electromagnetic force has *U(1)* symmetry, and using the complex numbers (*a*+ *bi),* it can define the up versus down spin of electrons and electromagnetic behaviors. The weak nuclear force, which describes radioactive decay and participates in nuclear fission and fusion, has other measures like "isospin" which require using *SU(2).* The strong nuclear force, which holds together the sub-elements of protons and neutrons (quarks) with three types of "color" charge, requires *SU(3)* symmetry.

Symmetry is essential in physics because, following Noether's theorem, each continuous symmetry that follows the principle of least action has a corresponding conservation law. The previous examples of time, spatial, and rotation invariances leading to the conservation of energy, momentum, and angular momentum (Figure 5-20) are examples of collectively shared external symmetries. In contrast, the *U(1), SU(2),* and *SU(3)* are intrinsic symmetries. For example, electron spin does not align to external orientations, but rather internal orientations. In modern particle physics, the *U(1), SU(2),* and *SU(3)* symmetries each have a conservation law corresponding to different fundamental physical forces and elementary particles.[132]

Example 7.6
Elementary Particles

Elementary particles in the Standard Model compose matter and forces (except gravity), and can have variations, such as anti-particles.

Bosons: (Force Carriers)

- *Photon*
- *W boson*
- *Z boson*
- *Gluon*
- *Higgs*

Fermions: (Matter)

- *Quarks: (up, down, charm, strange, top, bottom)*
- *Leptons: (electron, muon, tau,* $\textit{neutrino}_e$, $\textit{neutrino}_m$, $\textit{neutrino}_t$*)*

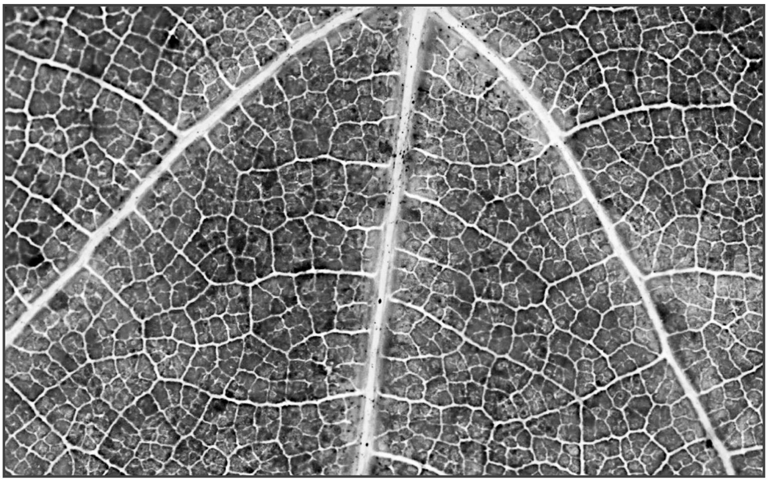
Symmetry	Conservation	Force	Force Particles	Particles Experiencing
U(1)	*Electric Charge*	*Electromagnetism*	*Photon*	*Electrically charged*
SU(2)	*Weak Isospin*	*Weak Force*	*W boson, Z boson*	*Quarks and Leptons*
SU(3)	*Color Charge*	*Strong Force*	*Gluon*	*Quarks and Leptons*

Figure 7-35 Symmetries in Standard Model of Particle Physics

Summary

Symmetry provides a foundational tool to identify patterns in systems. Symmetry can be used as a tool to model the structure and organization of systems with many pieces, from crystals, to flowers, to particle physics. Symmetrical and approximately symmetrical arrangements commonly form in natural systems to achieve optimal packing or to follow the principle of least action. While it can seem that nature follows a perfect underlying symmetry, there also exists asymmetry and randomness in the world. The next chapter will explore how chaotic processes can lead to new kinds of fractal patterns, extending symmetry to more complex systems.

Chapter 8 Fractals



Example 8.1
Fractal Leaf

The structure and pathways of a leaf form a pattern that repeats from small to large scales, providing a strong, efficient, and sensitive form.

Fractals give insight to a common pattern of systems: scalability. Unlike translational or rotational symmetries that describe changes over distances or rotations that repeat the same result, fractal symmetries describe transformations that repeat over different sizes of scale, as shown in Figure 8-1. A system with a fractal symmetry will have a repeating pattern that spans from small to large scales.

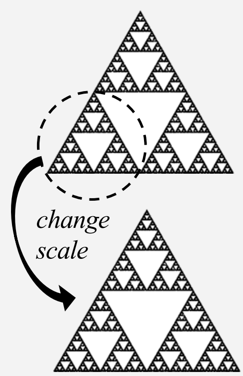
$$S:\{X \xrightarrow[Scale]{} X\}$$

Figure 8-1 Equation for Fractals

Fractals present a useful tool for systems science by uncovering patterns spanning small to large scales and identifying unifying rules that apply to both components and collections. Fractal patterns can be seen in the shape of crystals, fluids, geology, and plants, as well as ecological and socioeconomical trends. Fractals are deeply related to chaos theory and provide insights into the distribution of outcomes from chaotic dynamics in systems as widespread as random particle motion, stock prices, and earthquakes. Power laws, which describes how a given measures change across ranges of scale, is a very general and powerful way fractals can be applied to study natural systems. Altogether, fractals provide an important way to study structure, connectivity, and complexity of systems.

Example 8.2
Fractal Triangle

Triangles can be nested into a fractal symmetry. Changing the scale produces the same pattern.



Fractal Geometry

Fractals can be demonstrated through the combination of polygon units. By stacking three triangles together, it is possible to create a larger triangle, which can then be used as a unit to stack into an even larger triangle. Continuing iterations of this pattern create the Sierpiński triangle fractal.[133] Similar fractal stacking arrangements can be constructed with squares or pentagons, as shown in Figure 8-2. These polygon fractals can continue for a finite number of iterations or be repeated infinitely in a never-ending recursion.

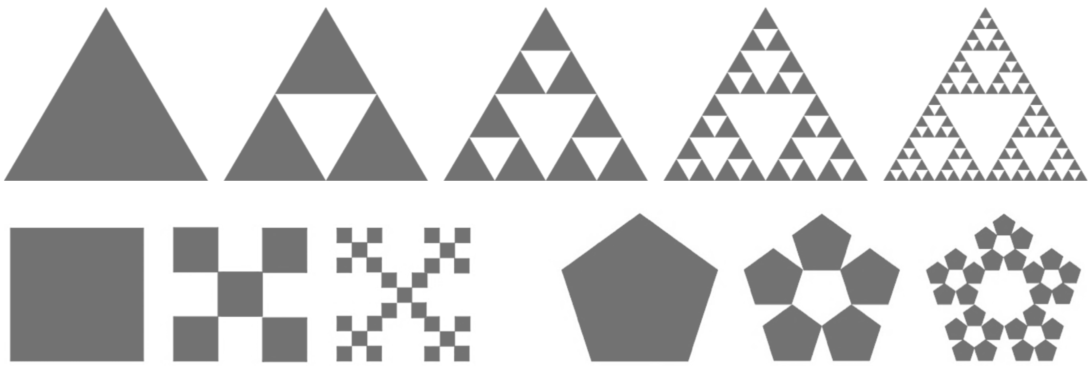
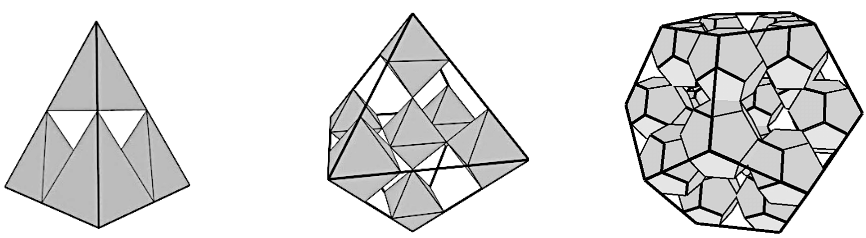


Figure 8-2 Triangle, Square, and Pentagon Fractals

Fractal geometry can be constructed in a similar fashion with 3-D polyhedrons. For example, multiple tetrahedrons can be stacked to form a large tetrahedron, which can be used as a unit for further iterations. Fractals can be similarity made with octahedrons and dodecahedrons, as shown in Figure 8-3. Volumetric fractals can occur in crystal lattice structures, where similar patterns repeat from small to large scales. Most crystals have periodic cubic and tetrahedral lattices, while quasicrystals can form aperiodic icosahedral arrangements.[134]



Tetrahedron Fractal *Octahedron Fractal* *Dodecahedron Fractal*

Figure 8-3 Regular Polyhedrons with Fractal Stacking

Another method to create a fractal is to repeatedly divide the perimeter of an object into smaller segments. The Koch snowflake, for example, is created by dividing the perimeter of a triangle into smaller protruding segments to form a six-pointed star, then continuing to subdivide the perimeter into a snowflake-like pattern, as shown on Figure 8-4. Surprisingly, the Koch snowflake will have a perimeter length that grows longer and approaches infinity as this fractal iteration is continued, even though there is only a finite surface area bounded.[135] The changing perimeter length with smaller line segments relates to the coastline paradox, which is the intriguing result that the measured length of a coastline can differ depending on the scale of measurement used.[136]

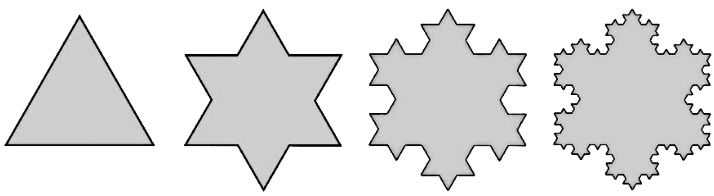
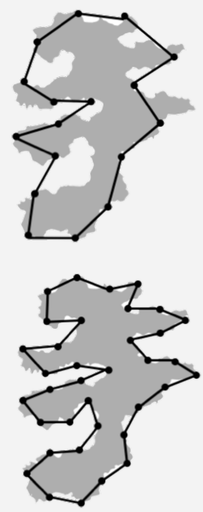


Figure 8-4 Koch Snowflake

The Koch snowflake can be extrapolated into three dimensions. This is accomplished by starting with a star tetrahedron and iteratively adding smaller protruding star tetrahedrons to each vertex point, as shown in Figure 8-5. Carried out to a high degree, this fractal will increase and approach an infinite surface area, even though there is a finite volume enclosed. Biological systems often form in a similar protruding fashion to maximize surface area. For example, protrusions on the outer membrane of cell, called microvilli, maximize surface area for diffusion to occur. The airways of the lungs also have iterative, and fractal-like, structures called alveoli that maximize the surface area for oxygen absorption.[137]

Example 8.3
Coastline Paradox

The coastline paradox describes that the coastline of a landmass does not have a well defined length and can change based on resolution. Coastline length typically increases with a more detailed resolution that has smaller distances between adjancent measurements.



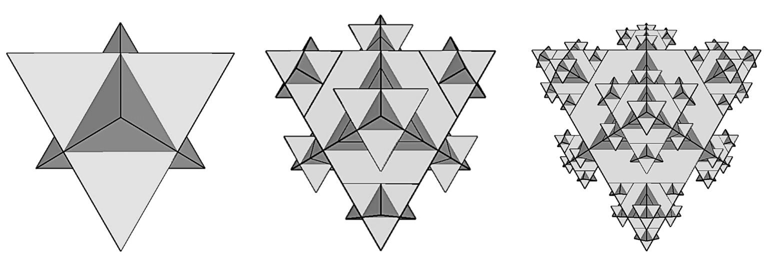


Figure 8-5 3-D Koch Snowflake

> **Example 8.4**
> Hausdorff Dimension
>
> The equation for the fractal dimension *D*.
>
> $Measure = Scaling^{D}$
>
> $$D = \frac{\log(Measure)}{\log(Scaling)}$$

The Hausdorff dimension *D*, or fractal dimension, describes how scaling a geometric system will change length, area, volume, or other measures by a specific power. A point has a fractal dimension $D = 0$, a line has $D = 1$, a square has $D = 2$, and a cube has $D = 3$. When scaled by two units, a 1-D line's length will grow by a factor of two $(2^1 = 2)$, a 2-D square's area will grow by a factor a four $(2^2 = 4)$, and a 3-D cube's volume will grow by a factor of eight $(2^3 = 8)$, as displayed in Figure 8-6. In most common shapes like polygons and polyhedrons, the fractal dimensions are whole numbers that equal the topological dimension, like *1*, *2*, or *3*.

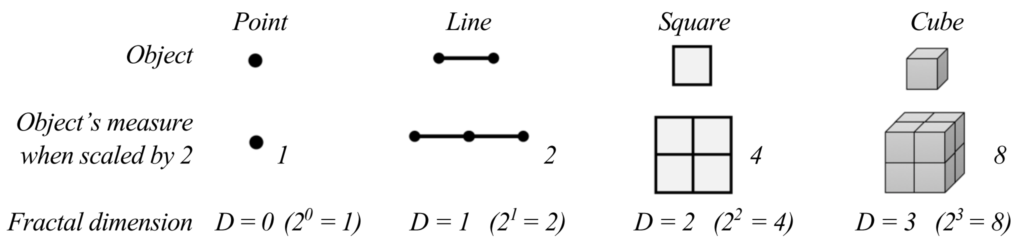


Figure 8-6 Fractal Dimensions and Scaling

Particularly interesting fractal geometries occur when the Hausdorff dimension of an object does not equal the spatial dimension in which the object resides. For example, the Sierpiński triangle resides on a 2-D plane, but the object has a fractal dimension of $D = 1.58\ldots$, as the pattern's area only gets three times larger when doubling the side length, as shown in Figure 8-7. In contrast, a regular triangle has a fractal dimension of $D = 2$, which is equal to a plane's topological dimension. Greater fractal dimension generally relates to more coarseness. For example, the fractal dimension of the Koch Snowflake increases with more iterations. In cell membranes, greater fractal dimension corresponds to increased surface area to exchange nutrients.[138]

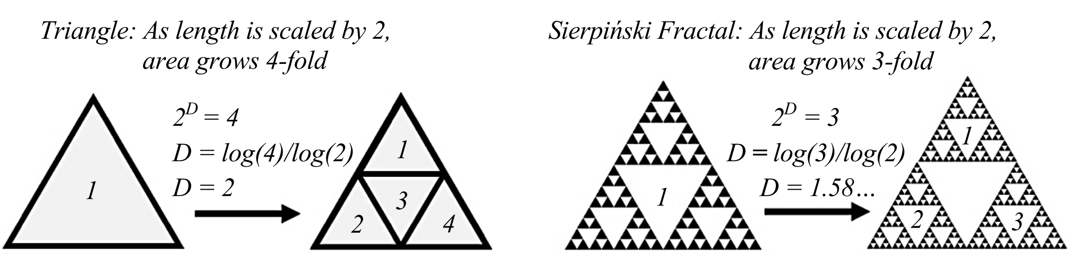
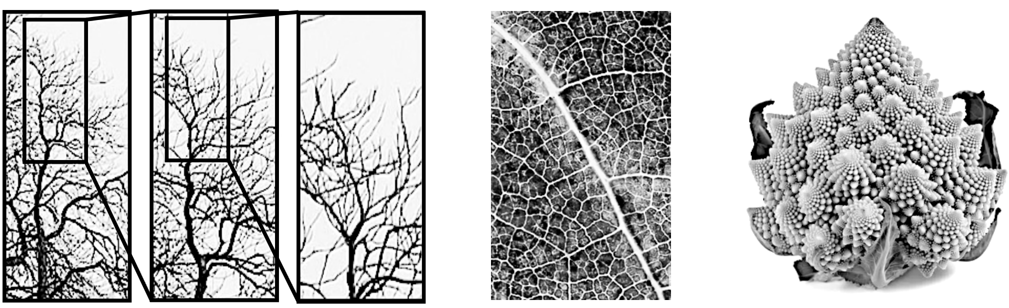


Figure 8-7 Scaling a Triangle and a Sierpiński Fractal

Fractals in Living Systems

Living systems often possess fractal-like structures, processes, and patterns to organize across a range of scales.[139] Cells group to form tissues, which then combine to form muscles. Tissues and muscles form organs, which then make up larger anatomical systems. Through synchronized organization across multiple scales, the human body acts as a single functioning system. In a similar way, individual organisms can act as the base units that make up the larger populations in an ecosystem. These populations relate through evolutionary patterns at the scale of the Earth's broader biosphere. The nested patterns of living systems resemble fractals in their ability to establish complex and coherent patterns across small to large scales.

Fractals can be observed in the shape, or morphology, of plants and animals. For example, small to large tree branches tend to repeat patterns at different scales, as shown in Figure 8-8. Fractal symmetries also occur in the branching patterns of veins, arteries, and the nervous system.[140] The growing pattern observed in Romanesco broccoli is symmetrical across scales, as each spiral cone resembles the whole plant. One useful and efficient functionality of fractal patterns are that they allow space-filling forms to continuously grow from small to large scales through a single repeating pattern. Additionally, fractal patterns can be optimal solutions to maximize the surface area to exchange nutrients and simultaneously minimize the transport distance and times across the living system.[141]



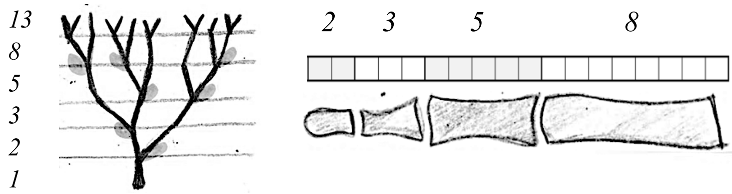
Tree Branches *Leaf Structures* *Romanesco Broccoli*

Figure 8-8 Fractal Patterns in Plant Morphology

Another pattern related to fractals found in biology is the Fibonacci sequence. The Fibonacci sequence {*0, 1, 1, 2, 3, 5, 8, …*}, introduced in Chapter 6, is created by summing the two previous numbers to create the next number in a self-similar fashion, written as ($x_n + x_{n+1} = x_{n+2}$). The recursive Fibonacci sequence is relevant to biology and arises in the number of clockwise and anti-clockwise spirals in pinecones, pineapples, and sunflowers.[142] The Fibonacci sequence is seen in the splitting of some plant stems and approximated in the ratio of the lengths of finger bones, shown in Figure 8-9.[143]

Fibonacci Sequence: $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots\}$

Self-Similar Sequence ($x_n + x_{n+1} = x_{n+2}$): $1 = 1 + 0,\ 2 = 1 + 1,\ 3 = 2 + 1,\ 5 = 3 + 2, \ldots$

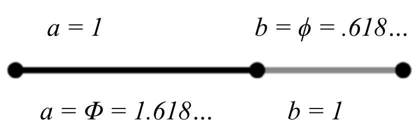


Plant Splitting *Human Finger Bones*

Figure 8-9 Fibonacci Numbers and Morphology Patterns

As the sequence increases, the relationship between Fibonacci terms approaches the golden ratio. For example, the ratio of the Fibonacci numbers of *89* and *55*, (*89/55 = 1.61818…*) is very close to the golden ratio ($\Phi = 1.61803\ldots$). The golden ratio is irrational, so it can never be perfectly expressed by a ratio of whole numbers, but adjacent Fibonacci terms approach the ratio when increasing.

The golden ratio is one of the simplest fractal patterns, as it requires one division within a line segment. The golden ratio recursively splits a line segment so that the ratio of the small part to large part, a / b, is equal to the ratio of the large part to the whole line segment, $(a + b) / a$. Another recursive aspect of the golden ratio is seen in the continued fraction representation. The golden ratio equals one plus one, divided by one plus one, divided by one, and so on, as shown in Figure 8-10. Since the number one is the lowest whole number, the golden ratio requires the most iterations to improve accuracy through a continued fraction and can be considered the most irrational number.[144]



$$\Phi = 1.618\ldots = 1 + \phi = 1/\phi$$
$$\phi = 0.618\ldots = \Phi - 1 = 1/\Phi$$

$$\Phi^2 = \Phi + 1 \qquad \phi^2 = 1 - \phi$$
$$\Phi^n = \Phi^{(n-1)} + \Phi^{(n-2)} \qquad \phi^n = \phi^{(n-2)} - \phi^{(n-1)}$$

$$\Phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

Golden Ratio: $a / b = (a + b) / a$ *Common Formulas* *Continued Fraction*

Figure 8-10 Golden Ratio

Approximations of the golden ratio are often found in the shapes of biological systems. For example, if the height of a person is treated as *1* unit, the length from their feet to their belly button is approximately the ratio $1/\Phi$. Going further, the length of the arm is the ratio $1/\Phi^2$, the length of the foot is $1/\Phi^4$, and the length of the hand is $1/\Phi^5$.[145] These ratios and other examples are depicted in Figure 8-11. Not every human body has the same proportions, but on average, many features approximate the golden ratio. Other ratios and fractal scaling laws, beyond the golden ratio, also apply to biological systems.

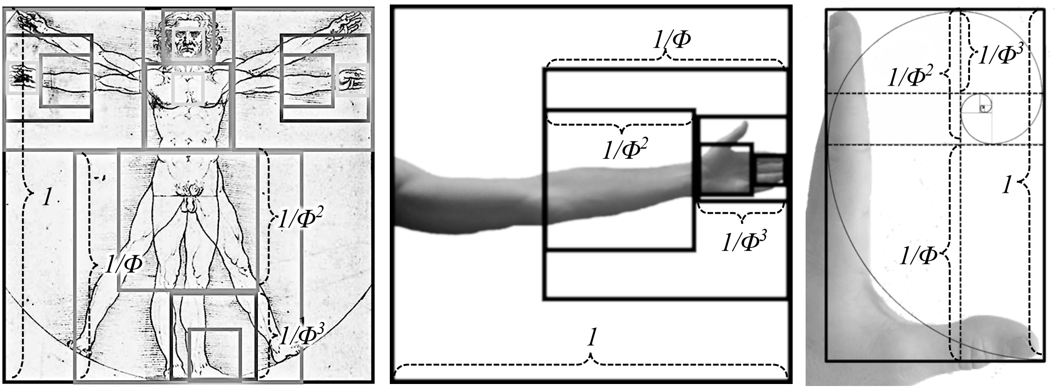
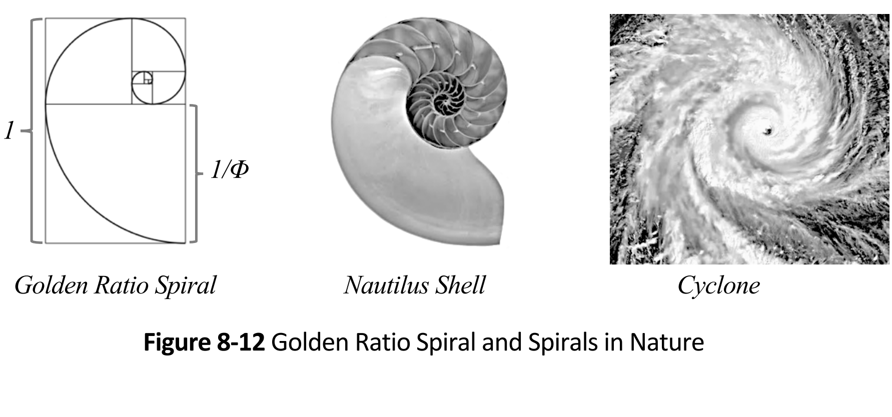


Figure 8-11 Golden Ratio in Human Body

Spirals, which follow a similar pattern when magnifying or minimizing scales, are another common type of fractal geometry observed in nature. In the golden ratio spiral, each quarter rotation around the center makes the curve move farther away from the center by the golden ratio. Approximates to golden spirals can be observed in natural patterns, such as the human ear or seashells.[146] Spirals can also have other ratios and form the broader class of logarithmic spirals. Spirals are a common pattern in living systems as well as physical systems, like weather patterns, that span many scales of size.



Golden Ratio Spiral *Nautilus Shell* *Cyclone*

Figure 8-12 Golden Ratio Spiral and Spirals in Nature

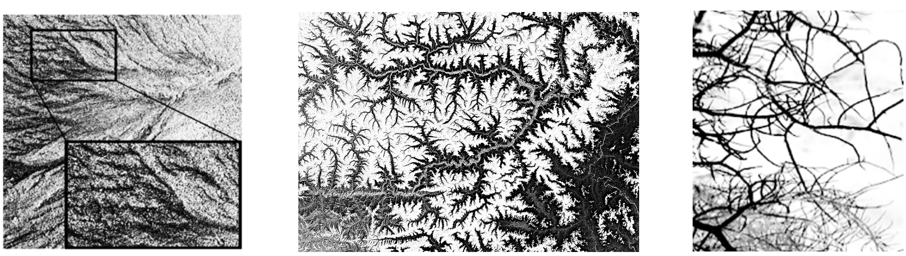
Dendritic Formations

Dendrites are a common fractal pattern that arises in natural systems. Dendrites look like growing branches that repeat from small to large scales. Dendritic patterns often come about as liquids or gases transform into a solid, such as molten metal solidifying.[147] Dendrite branches can be seen in snowflake crystals and in the electric arc paths of lighting, as shown in Figure 8-13. Another process that creates dendrite structures is diffusion-limited aggregation (DLA). DLA structures are created when particles moving in random motion latch onto a stationary seed particle. Electrodeposition, which is a process to solidify freely moving particles to a charge surface, creates materials that closely resemble DLA patterns.[148]

Ice Crystal Dendrites *Lighting Dendrites* *DLA Dendrites*

Figure 8-13 Dendritic Formations

Fractal dendrites can be observed in geological and biological systems. For example, glaciers carve dendritic forms in mountain ranges, as shown in Figure 8-14. As water flows down mountains through the principle of least action, drainage systems carve scale-invariant networks that resemble dendrites.[149] Dendrites can also be seen in the root systems of plants that split into small branches to efficiently absorb water and nutrients. These fractal dendrites are efficient patterns to support small to large scale resource flows.



Sand Formation *Mountain Rivers* *Root System*

Figure 8-14 Dendrites in Geophysical and Biological Patterns

The brain is another biological system where fractals and dendritic patterns support structure and function. Neurons, or brain cells, have protruding branch-like structures, which are also coincidentally called dendrites. Neural dendrites collect signals that are processed in the cell nucleus and influences if an electric pulse is sent as an action potential through the neuron's axon to other brain cells. A diagram of a neuron's structure is depicted in Figure 8-15. Neural dendrites allow brain cells to connect with one another and form an intricate network of relationships to process information. The neural network is not random, but instead organized in specific pathways to enable information processing and biological functions. Organizing a neural network is an immense task as there are about 86 billion neurons and trillions of synapses in a human brain.[150]

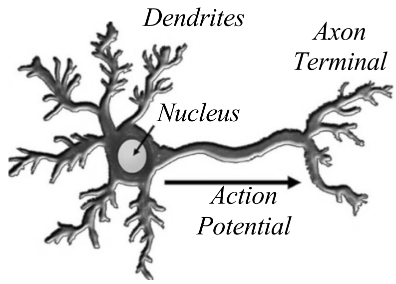


Figure 8-15 Neuron Structure

Fractals, along with other organizing patterns, are suspected to occur in animal nervous systems and neural networks to support functionality. Fractal geometry can be seen in the complex and densely interconnected structures of dendrites in neural networks, like Figure 8-16. These fractal patterns in neural networks can support the creation of nested systems of hierarchies, high degrees of sensitivity, and local-to-global organization.[151] Fractals can also serve as useful patterns from an informational perspective by being able to efficiently input, store, and output large amounts of information by repeating rules over multiple scales. Biological systems even utilize self-similar logic and fractal-like information systems that can optimize both micro and macro stimuli.[152]

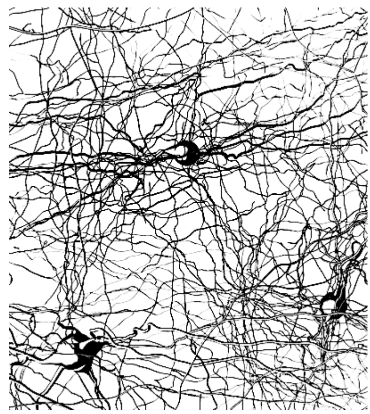


Figure 8-16 Neural Network

Bifurcating branching patterns provide a crude analogy for how a fractal organization can support the formation of the brain. This fractal pattern, shown in Figure 8-17, first splits into two opposing sections, similar to the dual hemispheres of the brain. These branches are then further divided into subsections, and even smaller subsections, that become increasingly detailed. Neural networks observed in the brain have much more specified and complex patterns, but this example shows how fractal patterns can support micro-to-macro connectivity and collective optimization of networks.

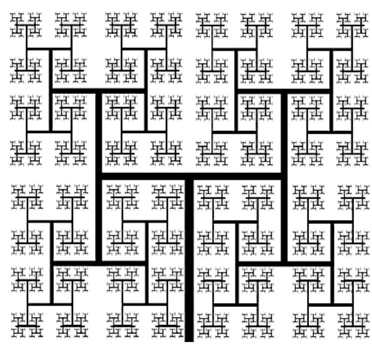


Figure 8-17 Fractal Branching

Chaos and Fractals

Fractal patterns often arise in random and chaotic systems. For example, the "chaos game" begins by placing a dot anywhere in a triangle. Next, new dots are placed halfway between a randomly chosen vertex of the triangle and the previous dot. At first, the dots appear randomly placed, but a fractal pattern eventually forms after many iterations. Chaos games can be performed with different polygons and placement ratios to produce a wide array of fractal patterns, some shown in Figure 8-18.

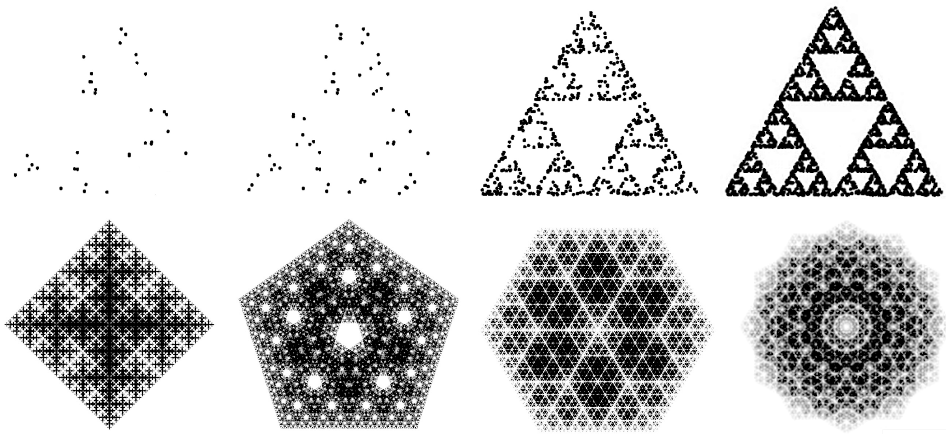


Figure 8-18 Chaos Game Fractals in Regular Polygons

Random walks are also deeply connected to fractals. A random walk marks a path undergoing random displacement changes within certain bounds (e.g. *-1* to *1*). Figure 8-19 shows a 1-D random walk, with a random vertical change and constant horizontal change, as well as a 2-D random walk, with random changes to both the vertical and horizontal locations. After many iterations, a random walk path creates a pattern that is scale-invariant.[153] This means the random walk will have the same fractal dimension and the other scale-free measures of average displacement from small to large scales.

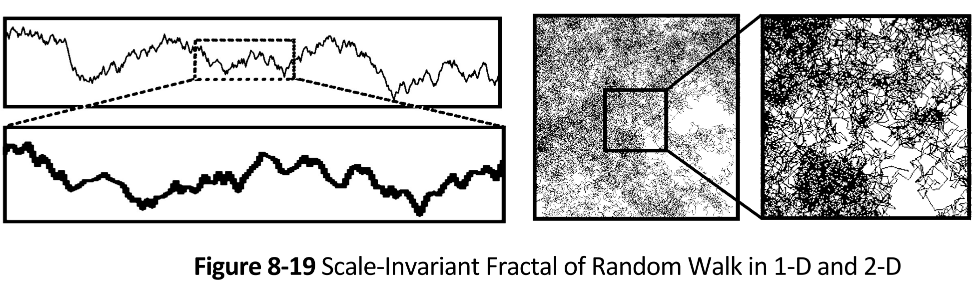


Figure 8-19 Scale-Invariant Fractal of Random Walk in 1-D and 2-D

The collective average of many random walk paths produces the emergent higher-level result of smooth diffusion. In fluids and gases, each molecule follows an approximate random walk, called Brownian motion, caused by many chaotic collisions. When considering a large number of random walks, concentrations of particles will tend to evenly spread apart and follow the diffusion equation, as shown in Figure 8-20.[154] While diffusion is often pictured as a smooth process, it can also be modeled through the collective behavior of many random walks, each with a fractal symmetry.

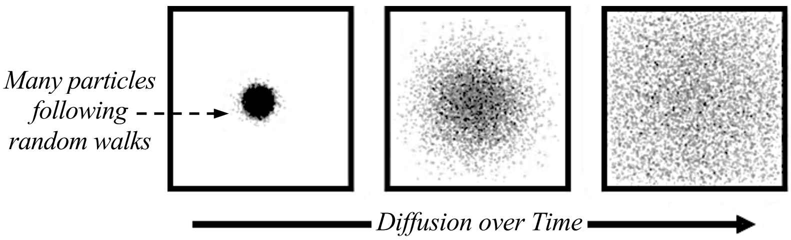


Figure 8-20 Diffusion of Random Walk Collections

Random walks can be applied to surfaces to create fractal landscapes. Between the adjacent points, the surface height changes by a random degree which creates irregular bumps and dips. Variations of this process result in patterns that resemble mountain ranges and other geologic formations, as shown in Figure 8-21. This random walk surface is influenced by different boundary perimeters and the maximum height variations allowed per step.

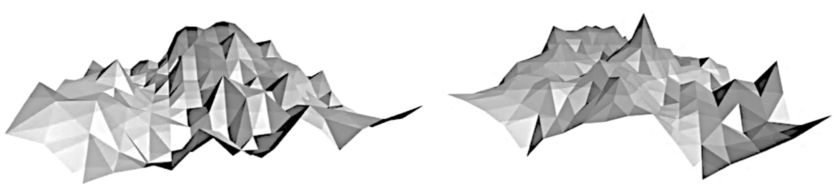


Figure 8-21 Random Walk Fractal Landscapes

Fractals can be used to analyze volumetric distributions in systems. For example, fractal distributions arise in water vapor clouds, which are rough and lumpy yet also have similar variations across small to large scales. Expanding further, fractal analysis can help analyze the network-like distribution of matter and dark-matter (a form of matter that interacts with gravity, but not electromagnetic forces) in the universe.[155] There is even a fractal pattern in the cosmic microwave background temperature across space.[156]

Chaotic Attractors

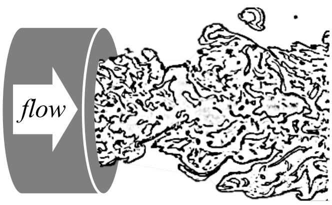


Figure 8-22 Fractals in Fluid Turbulence

Dynamic systems with nonlinear rates of change can drive the formation of fractal patterns. The chaotic turbulence of fluids, for example, exhibits a fractal dimension where similar patterns exist across multiple scales of size. More generally, the equations of fluid dynamics are scale-free, and patterns often repeat from small to large scales. Nonlinear systems can be highly sensitive to initial conditions and can lead to chaotic patterns that cannot be efficiently predicted with high degrees of accuracy. However, fractals can provide some insight into understanding the chaotic distribution of outcomes.

A pioneering mathematician in the late 19th and early 20th century that provided essential groundwork for chaos theory was Henri Poincaré, who introduced new methods for celestial mechanics when trying to solve the chaotic three-body problem.[157] Poincaré used the concept of phase space in attempting to find patterns in these chaotic motions. Each point in phase space represents a possible state of a system, such as a certain position and momentum. When graphing the evolution of chaotic and nonlinear dynamical systems in phase space, new types of patterns emerged. Even though these phase space patterns usually never repeat and cannot be predicted with high degrees of accuracy across long time frames, the patterns can have interesting symmetries, like fractals. 20th century computers now have the ability to graph extremely large datasets, which provide a way to visualize hidden fractal patterns in the phase space of chaotic motion.

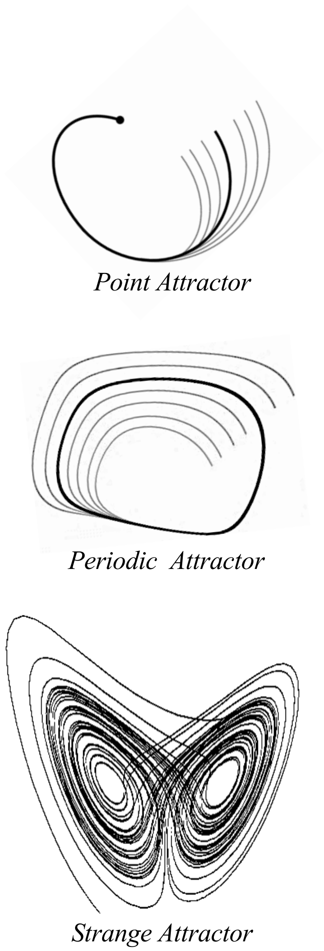


Figure 8-23 Chaotic Attractors

In chaos theory, an attractor is the tendency for a system to be drawn to a given pattern in phase space. There are many types of attractors, such as point attractors, cyclical attractors, and strange attractors. Point attractors settle towards a single state in phase space over time and periodic attractors settle into cyclical patterns. Both points and periodic attractors can be approximated by linear systems of equations. In contrast, strange attractors do not have any easily solvable or periodic result. For example, the Lorentz attractor pictured in Figure 8-23 describes fluid convection currents exposed to energy. This non-repeating strange attractor sometimes favors the left or right side, or suddenly switches. While seemingly random, this chaotic system has a fractal distribution of paths in phase space, where large to small bands of paths clump together, similar to the rings of Saturn.

One of the most famous strange attractors is the Mandelbrot set, named after Benoit Mandelbrot who popularized the concept of fractals in the book *The Fractal Geometry of Nature* published in 1982. The Mandelbrot set includes bounded values of c when iterating the nonlinear equation $x_{n+1} = x_n^2 + c$ beginning with the value $x_1 = 0$. In this equation, the constant c is a complex number ($c = a + bi$), where $i^2 = -1$. As multiple iterations of x are continued, some c values cause x to approach infinity, while others do not. For example, when $c = -1$, the series (0, -1, 0, -1, 0, ...) will be bounded and included in the Mandelbrot set. When $c = 1$, the series (0, 1, 2, 5, ...) will increase infinitely and thus be excluded from the Mandelbrot set. At increasing magnification, many intricate patterns appear and repeat over different scales, some of which are shown in Figure 8-24.

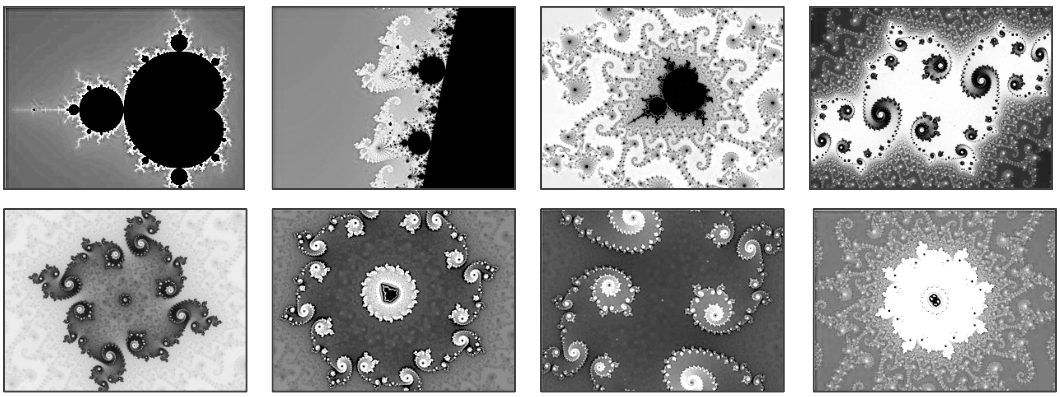
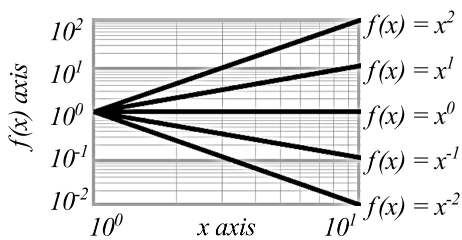


Figure 8-24 Magnifications of the Mandelbrot Set

Chaotic dynamics show up in many kinds of natural systems. Weather predictions more than about 10 days in the future is near impossible to perform with high degrees of accuracy due to the chaotic nature of the atmosphere.[158] The weather may follow general trends, such as more rain in winter and less rain in summer, but specific day-to-day weather conditions cannot be predicted with high fidelity. Chaotic systems are also present in modeling geologic formations, biological populations, and a variety of socioeconomic patterns like stock market volatility. While similar, chaos differs from pure randomness because there are some underlying patterns and relationships, rather than pure random noise. Chaotic systems are also subject to large and unexpected changes, like stock market crashes, and other macro-level events, like phase transitions, which happen much more frequently than would occur in a random model.

Scaling Relations

A very generalized way to apply fractals is through power laws. Power laws occur when an output is proportional to an input raised to a power, or fractal dimension D, written $f(x) \propto x^D$. For example, with a sphere's radius equal to x, its surface area is proportional to the square x^2, and its volume is proportional to the cube x^3. Power laws are scale-free, which means a relative change in one quantity gives rise to a proportional relative change in the other quantity, independent of the initial size of those quantities. For example, there is no preferred length, or units of measurement, where the relation to area (2-D) and volume (3-D) deviates from the power rule. Power laws graphed on logarithmic scales $(\ldots\ 10^{-2}, 10^{-1}, 10^{0}, 10^{-1}, 10^{-2}, \ldots)$ produce straight lines with constant slopes equal to D, as shown in Figure 8-25.



x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125

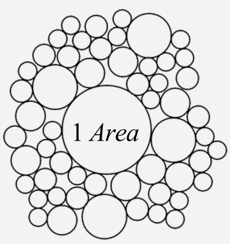
Figure 8-25 Power Laws and Logarithmic Scales

Many physics equations used to model nature are based on power laws. For example, gravitational and electric forces are inversely proportional to the square of the distance, following $force \propto distance^{-2}$. Similarity, the intensity of light beams emitted from a point source reduces over distance following the inverse square law, $intensity \propto distance^{-2}$. The restoring force wave equation follows a $D = 1$ power law as $force \propto -distance^{1}$. These power laws are all scale-free and exhibit consistent behavior across small to large distances.

Power laws even arise in linguistical systems. For example, a word's frequency of appearance in a written text is typically inversely proportional to its rank of usage. This means that the 2nd ranked word is about ½ as likely as the 1st, and the 4th ranked word is ¼ as likely. High rank words like *"the"* and *"of"* are used very often, but less frequent words like *"chair"* are used much less. This inverse proportionality, called Zipf's Law, is approximately matched in text from all human languages.[159]

Example 8.5 Power Law Distribution

In the figure below, the circle count to circle area follows a power law relation, $Count = Area^{-2}$ Power laws lead to structures across scale, rather than arrangements of units the same size.



Power Law Circles:

Count	*Area*
1	1
4	½
9	⅓
16	¼
25	⅕

Power laws are also prevalent in biological systems. For example, an animal's metabolic rate is approximately proportional to its mass to the ¾ power ($\textit{metabolic rate} \propto \textit{mass}^{3/4}$). Small animals, like mice, tend to have much higher metabolic rates than large animals, like elephants. The ratio ¾ is suspected to commonly arise because it is an optimal solution for a branching network to minimize the required transport distance to service a volume.[160] Power laws occur in other biological processes, like heart rates, circadian rhythms, breathing intervals, and vocal patterns.[161] Power relations can even been seen in ecological patterns. Taylor's laws, which relate a measure's variance (closeness to an average) to the average ($\textit{variance} \propto \textit{average}^{D}$), can accurately describe population distributions, crop yields, and disease spread.[162]

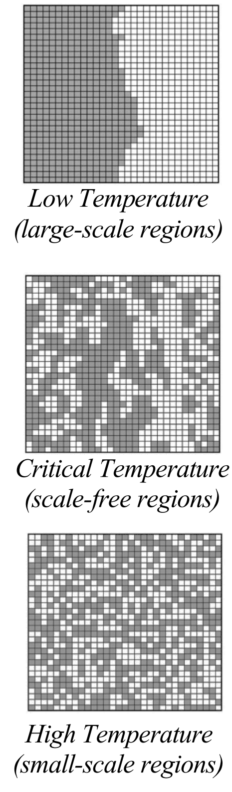


Figure 8-26 Scale-free Critical Points

Power laws can occur at the boundaries between phases, called critical points. The Ising model of magnetic diploes in a lattice shows this well. At low temperatures, the dipoles will align into an ordered phase with large regions of positive or negative charges, shown as grey and white boxes in Figure 8-26. High temperatures cause the dipoles to move into a random disordered phase, with only small regions of like charges. The critical state exists at the boundary of order and disorder, where there are both small and large regions of like charges that follow a scale-free power law. It is also suspected that vastly different systems and particle ensembles can have very similar critical point behaviors that are universally shared.[163]

Many kinds of complex systems are suspected to have a tendency towards critical states that do not require fine-tuned initial conditions. This so-called "self-organized criticality" occurs when systems have attractors that enable the system to tune itself towards criticality. A commonly cited example is that when dropping sand grains into a pile with a degree of randomness, the sizes of the avalanches that occur organize into a power law.[164] It is suspected that earthquakes, financial markets, and other types of complex systems can be attracted to produce scale-free criticality. Critical behavior and phase transitions may also help explain the patterns in complex networks, such as neural network activity.[165]

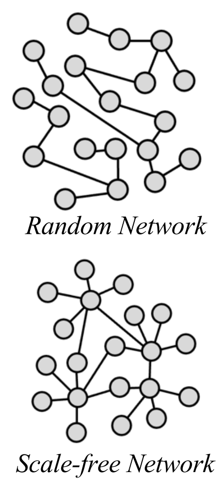


Figure 8-27 Scale Free Networks

Networks are another example of complex systems with scale-free relations and critical behavior. Unlike randomly generated networks, where most nodes have about the same number of connections, scale-free networks follow power laws and develop both hubs with large number of connections and nodes with few connections, as shown in Figure 8-27. Internet website links resemble scale-free patterns, with some websites serving as large hubs.[166]

One potential explanation for how scale-free networks arise is preferential attachment. Preferential attachment models a process where a node has a higher preference to attach to nodes that already have many connections. For example, people in social networks are often drawn to follow and connect with large influencers that already have many exiting connections. By following preferential attachment rules, networks can self-organize to have large hubs of activities that have scale-free power laws, deviating from what would be expected in a random network.[167]

While scale-free power laws commonly occur in natural systems, there are limitations. One limitation is that measured data of natural phenomena, especially complex behavior, may have variations or outliers that does not perfectly fit power laws. Power laws of complex systems often provide approximations rather than providing an exact result. There are also other scaling relations where *D* is not perfectly fixed and changes over scales to provide a better fit for real-world data. Even when not perfectly scale-free, complex systems often follow long-tailed functions, where low-probability configurations—like large network hubs—occur much more often than what would be expected from a random distribution. Long-tailed functions can lead to much higher chances of large deviations, like large earthquakes and ecological collapses, compared to what would be expected following random fluctuations around an average.

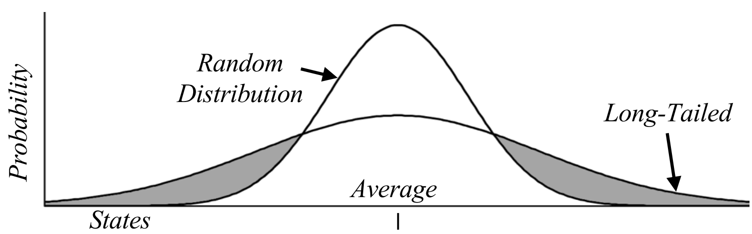


Figure 8-28 Long-Tailed Functions

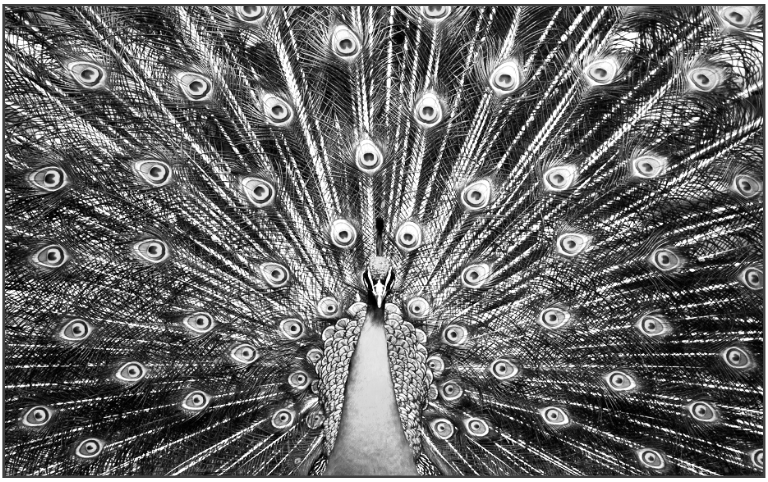
Another limitation is that power laws are typically only valid within a given domain. For example, the ratio of animal mass to metabolic rate doesn't apply to atomic scales or planetary scales, because there are no animals of those sizes. Physics provides a more formal description of limiting domains through effective field theories, which introduce particular energetic cut-offs, thereby defining the scale in which the theory is valid. For example, general relativity simplifies to Newtonian gravity, with cutoffs at low speeds compared to light. Similarity, the electromagnetic force and the weak force emerge as an effective field theory when setting cutoff limits below the energy level of the electroweak force, which is a unified model where the electromagnetic force and weak force are unified. A hypothetical unified field theory would define common relationships that span all energy and spacetime scales—quantum to cosmic—but currently, only inconsistently effective field theories have been found to model nature within specific domains.

Importantly, many models of nature are scale-dependent. In quantum field theory, a scale-dependent behavior is observed in the model of the strong force interaction. The strength of the strong force between quarks and gluons depends on the energy scale at which it is probed. This property is known as asymptotic freedom, where the strong force weakens at high energies and becomes stronger at low energies. The behavior at different scales is crucial in describing the properties of subatomic particles and the dynamics of the early universe. Scale-dependent patterns are essential to determining why certain phenomena occur at specific scales and not others, such as the size of atoms, molecules, planets, stars, galaxies, and galactic clusters. Science uses a combination of scale-free laws and scale-dependent laws to model nature.

Summary

Fractals can be observed in many systems and provide patterns for how smaller sections relate to larger collections. Fractals arise in the random walk pattern of atomic motion, chaotic attractors of nonlinear equations, dendritic crystal growth, biological morphology ratios, geophysical patterns, and perhaps even in the distribution of matter in the universe. Fractals connect to many attributes of systems, such as least action, chaotic dynamics, nested hierarchies, and local-to-global optimization, which provide an important conceptual foundation for complexity and connectivity.

Chapter 9 Order



Example 9.1
Peacock Patterns

The feathers of a peacock are highly ordered. These intricate patterns attract mates and influence evolution.

Complex systems in nature and society express high degrees of order. Order is here defined to increase when the change of entropy is less than zero, which relates to concentrating thermodynamic energy and reducing the number of microstates. Entropy measures can also be applied to information theory following Claude Shannon's definition. Shannon entropy is the average symbolic information needed to define an object's states, which includes macro-level states like if an electrical transistor is on or off. Systems in nature, such as DNA, neural networks, and computers, can organize and reduce both energetic and informational entropy.

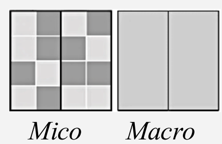
$$S:\{\Delta Entropy < 0\}$$

Figure 9-1 Equation for Organization

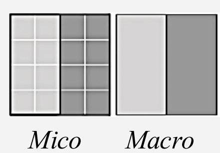
The tendency to increase order can only occur in systems open to the flow of energy. Following thermodynamics systems closed to energy and matter tend to either increase in entropy or maintain a constant entropy. A critical caveat is that open energetic systems can reduce entropy by using energy in controlled ways. For example, biological systems are able to maintain local decreases in entropy by employing chemical reactions that harness inputs of energy. When powered by processes that are not perfectly efficient, decreasing the entropy of subsystems can only be allowed by (and can feed off) increasing the total entropy of the universe.

Example 9.2 Order

Disorder occurs when the same macrostate can arise from many possible microstates, as shown below.



Ordering a system creates a less likely macrostate.



Entropy Reduction

> **Example 9.3**
> Entropy Definitions
>
> In thermodynamics, the change of entropy in a open system relates to to the exchange of heat Q at temperature T:
>
> $$\Delta Entropy = \frac{\Delta Q}{T}$$
>
> Gibbs entropy relates to the sum (Σ) of the logarithim of p_x, the probability that the system is in the x^{th} state, and the constant k_b. Information-based Shannon entropy uses this same form, but removes the constant.
>
> $$Entropy = k_b \Sigma p_x \log(1/p_x)$$
>
> With equally likely states, entropy is proptional to the number of states.
>
> $$Entropy = k_b \log(states)$$

Natural systems have a remarkable ability to increase order and decrease entropy. This seems to refute the second law of thermodynamics that the entropy will be constant or increase in isolated systems. For example, a drop of dye in a glass of water will tend to disperse over time, rather than coalesce. While entropy cannot decrease in isolated systems, it can decrease in a system open to energy. An open system *(X)* can decrease in entropy if the external system *(U – X)* increases in entropy enough so that the total isolated system abides by the second law of thermodynamics, as shown in Figure 9-2. For example, a refrigerator uses energy to create a low entropy cold interior, but at the cost of expelling heat that increases the total entropy.

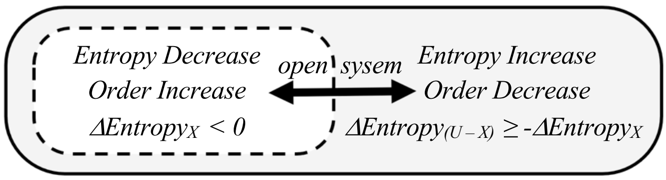


Figure 9-2 Order in Open Systems

Entropy relates to exchanging heat energy for useful work. The Carnot heat engine exchanges heat from a hot reservoir to a cold reservoir through a volume controlled by a piston. The engine can output positive work (W) as heat (Q) flows from the hot to cold regions, such as a steam engine. Conversely, heat differences between regions can be increased by inputting work, like a refrigerator. The Carnot cycle establishes the ideal upper limit of converting heat into work following ($W = Q_H – Q_C$), where the total change of entropy equals zero ($\Delta Entropy_H + \Delta Entropy_C = 0$). Practically, heat engines are not perfectly efficient, and entropy tends to increase. Despite inefficiencies, technologies like refrigerators can decrease entropy of specific open reservoirs by inputting work.

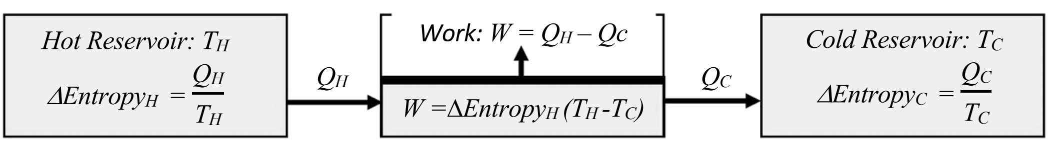


Figure 9-3 Carnot Heat Engine

Entropy can be defined through a statistical approach as the number of equally likely microstates that result in a given macrostate, written *Entropy* $\propto$ *log(states)*. For example, in a chamber with two sides that contain five grey particles and five black particles, there is just one state leading to the macrostate that the left side is filled entirely with grey particles, leading to low entropy. In contrast, there are ten possible states that lead to the macrostate that the left side includes three grey and two black particles, leading to higher entropy. These states, as well as flipping an equal chance coin, follow the binomial distribution. There is only one way to flip five heads in a row, but ten ways to flip three heads during five total flips. Separated particles will tend to mix and increase in entropy because there are more possible equally likely states for these arrangements to occur.

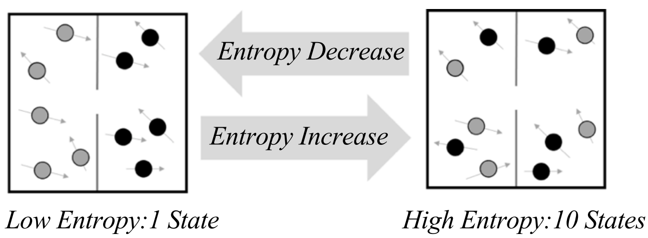


Figure 9-4 Entropy and Microstates

Entropy decreases as energy concentrates and the ability to perform work increases. Repelling electrically charged particles will have lower entropy and greater availability to do work when packed tightly together. Conversely, attracting gravitational objects will have lower entropy when moved apart. Oil and water have low entropy in a mixed state as there is greater potential energy to move into two distinct regions. While entropy tends to increase in isolated systems, energy can be used by open subsystems to lower entropy. Energy can be used to push repelling particles together, pull attracting bodies apart, or mix oil and water, thereby lowering entropy.

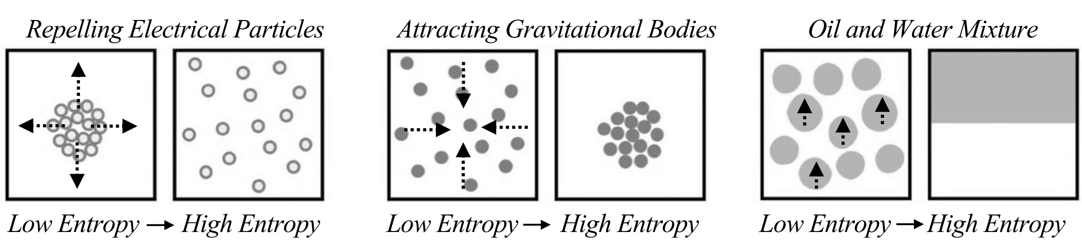


Figure 9-5 Entropy and Energy Dispersion

Powering Order

The Gibbs free energy G determines if a reaction will spontaneously occur at a given temperature T and pressure P. The Gibbs free energy depends on the entropy, the molecular kinetic energy U, and the volume V, following $G = -Entropy \cdot T + U + PV$. Exergonic reactions result in the negative change in free energy $(\Delta G < 0)$ and increase entropy at constant U and V. Exergonic reactions spontaneously occur and release energy. Conversely, endergonic reactions result in the positive change in free energy $(\Delta G > 0)$, are non-spontaneous, absorb energy, and reduce entropy at a constant U and V. Reactions at equilibrium have no change in Gibbs energy $\Delta G=0$.

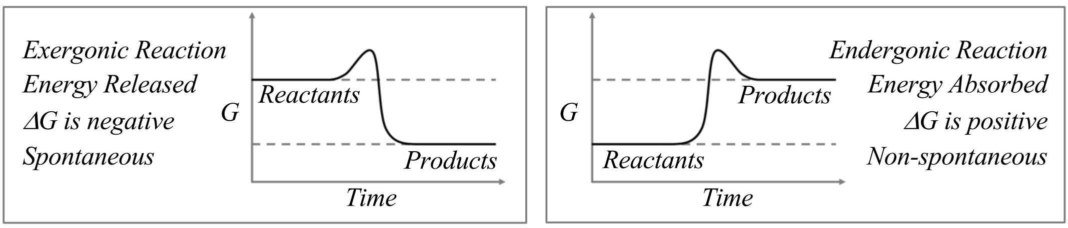


Figure 9-6 Exergonic versus Endergonic Reactions

Biological systems use a combination of exergonic and endergonic reactions to capture and release energy for useful work. A common chemical for carrying energy is adenosine triphosphate *ATP*. *ATP* molecules can react with water to release energy, alongside adenosine diphosphate *ADP* and phosphate P_i, following the exergonic reaction $ATP + H_2O \rightarrow ADP + P_I + Energy$. Conversely, energy can be used used an input, alongside *ADP* and P_i, to form *ATP*, following the endergonic reaction $ADP + P_I + Energy \rightarrow ATP + H_2O$. *ATP* molecules provide a method to exchange and transport the energy generated from exergonic reactions, such as breaking down glucose sugar, to power endergonic reactions that require energy, like muscle contractions or protein synthesis.[168]

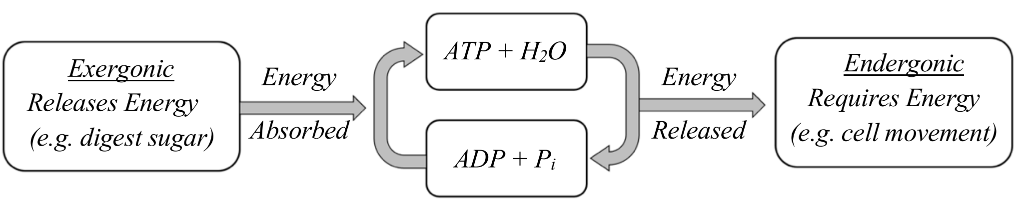


Figure 9-7 ATP Energy Exchange

Biological processes have numerous metabolic reactions that harness energy that can later be used to exert work and create order. One metabolic reaction is glycolysis, which converts the sugar glucose $C_6H_{12}O_6$ into lactic acid $C_3H_6O_3$, and in the process releases energy to make *ATP*. Cellular respiration also breaks down glucose, but uses oxygen O_2 and releases carbon dioxide CO_2 to create a more efficient process. Organisms can gather glucose from the environment and through glycolysis and respiration create *ATP,* which can be used to power the creation of highly ordered structures. Additionally, some cells evolved the ability to absorb energy from the Sun through photosynthesis to create glucose, which can be broken in further reactions to create energy. These metabolic reactions enable chemical energy to be converted into useful forms for work. Cellular respiration and photosynthesis also have reciprocal reactants and products that create a balancing effect on the atmosphere's carbon levels.

$$\underbrace{C_6H_{12}O_6 \rightarrow 2C_3H_6O_3}_{\text{Glycolysis}} \quad \underbrace{C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O}_{\text{Cellular Respiration}} \quad \underbrace{6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2}_{\text{Photosynthesis}}$$

Releases energy (2 ATP) — *Releases energy (36-38 ATP)* — *Requires sunlight energy*

Figure 9-8 Metabolic Reactions

Similar to thermodynamic entropy, ordering informational entropy requires the use of energy. Informational Shannon entropy relates to defining a symbolic message of a given probability and is measured through bits of data. The physical markers of all informational messages of data are limited by physical laws. One implication of the physical nature of information is that energy sources are required to write, store, and read data. Landauer's principle demands that at least energy $E \geq k_b \cdot T \cdot ln(2)$ is needed for an apparatus to create or erase one bit of data.[169] This means that energy is require to order information.

Ordering information is a commonly performed and highly useful action in biological and computational system. For example, information can be compressed in computers by running programs that reduce the number of bits required to convey a given message. This can save valuable memory resources and energy requirements for transmitting the message. DNA has also evolved to be highly data and energy efficient when instructing complex biochemical reactions. Higher-level cognitive processes often leverage the benefits of ordering data, simplifying messages, and creating shortcuts that allow less energy to process information.

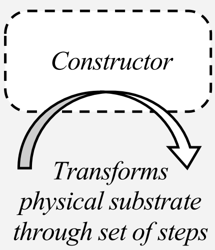
Autopoietic Systems

One question at the heart of scientific inquiry is what distinguishes a "living" system from a "nonliving" system. While the scientific community does not employ a strict definition of life, it is typically assumed that life needs to self-reproduce ordered and evolving structures. However, various machines can create order and self-reproduce, but are generally not considered "living." System theorists and biologists Humberto Maturana and Francisco Varela introduced the concept of "autopoiesis" in 1972 to broach this question.[170] Autopoiesis refers to a system capable of producing and maintaining itself by creating its own parts. In Greek, "auto" means "self", and "poiesis" mean "creation". Autopoietic systems are not only capable of self-reproduction, but also making and maintaining the very parts they are made of. Following Humberto's and Varela's theory, "life" occurs in the subset of physical systems that demonstrate autopoiesis.

Autopoietic, self-making, systems contrast allopoietic, or other-making, systems. An example of an allopoietic system is a bicycle factory. The factory uses energetic and material inputs to create a low entropy and ordered structure of a bicycle. However, this bicycle is different, or "other," from the factory. This contrasts an autopoietic system, such as a hypothetical living 3-D printer capable of producing and assembling its own machine parts. Self-making would include manufacturing all structures of the printer, including motors to run the printer and the computer chip with instructions for how to self-construct from simple inputs. This hypothetical printer could remove and install parts, as well as do any needed repairs by itself. While current 3-D printers have some of these abilities, no human-made machine has yet been classified as autopoietic or living. Instead, external processes accomplish some or all of these tasks, making these machines allopoietic.

Example 9.4
Constructors & Life

In constructor theory, which studies all possible transformations allowed in physics, life is a replicating constructor, which has the necessary or set of steps, or recipe, to make itself, as well as correct errors when replicating.



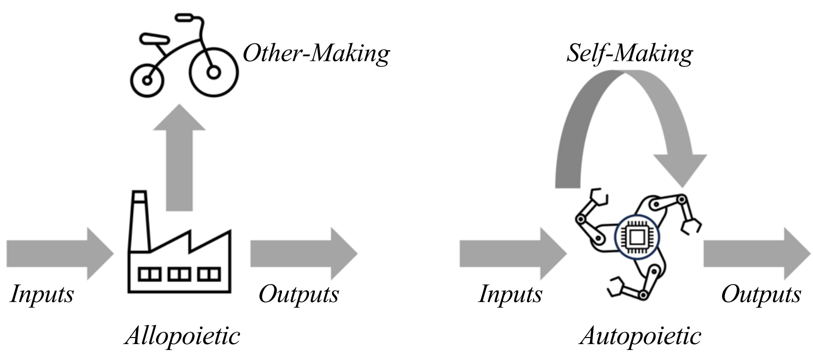


Figure 9-9 Autopoietic Systems

One important difference between allopoietic and autopoietic systems is the requirement for an external agent. In an allopoietic system, external processes are required to make components (like motors or computers) to build machines and software. Additionally, if the machine needs to be fixed, repaired, or replicated, external support (like humans) is typically needed. In contrast, an autopoietic system can make all the utilized components, self-reproduce, self-program, and make new replicas of itself. The self-referencing causality seen in autopoietic systems is similar to the chicken-and-egg problem. The process of life makes the components of life, and the components of life make the processes of life, with no external agent.

There is a meta quality to the information in a living system because it requires information needed to make itself physically. This contrasts current electronic computers where the informational processes—or software—is often not about the physical hardware itself. A hypothetical autopoietic computer would require software to be able to make its own hardware, repair itself, replicate itself, as well as operate mechanisms to accomplish these tasks. An autopoietic system's information is similar to an embedded metalanguage, as matter (e.g. DNA) marks instructions about how to alter matter itself.

In the early 1970's James Miller provided a theory of living systems that defined life as systems open to exchange of matter, energy, and information that self-organize and self-make through certain critical subsystems.[171] These subsystems included ingesting, storing, moving, and other actions to manipulate matter and energy. Other subsystems act on information, including data inputs, channels, memory, and decoders. Additional processes simultaneously pertain to matter, energy, and information, such as self-replication and managing the living system's boundary to the environment. Together, these subsystems work together to enable a living system to emerge that can self-organize, self-make, and self-replicate.

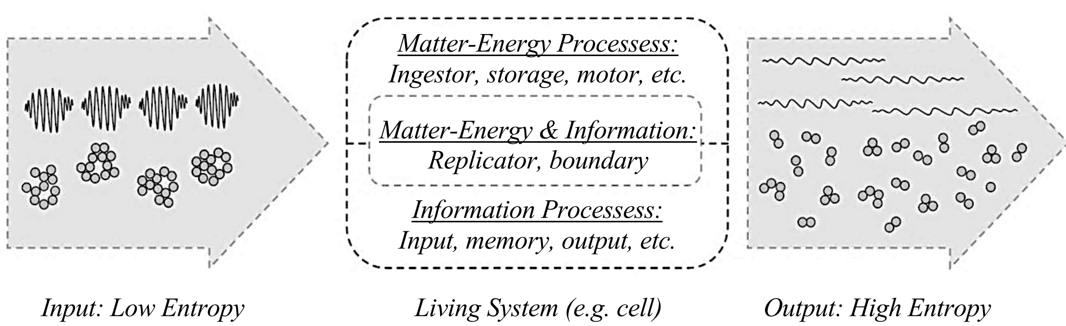
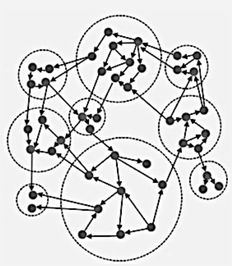


Figure 9-10 Living Systems Theory

Biochemical Organization

> **Example 9.5**
> Biochemical Networks
>
> Chemical reactions in living systems, like metabolism, protein synthesis, and replicating DNA, work in modular and interconnected subgroups. These networks are highly complex, nonlinear, and contain many feedback loops.
>
> 
>
> *Complex Network*

Living systems can self-make, and become ordered, by using energy in the environment. Sunlight is the primary energy source for life, which plants covert to chemical energy through photosynthesis. Animals then eat plants, which passes energy through the ecological food chain to the next group, or tropic level. In this process, approximately 10% of the total available energy is made accessible to the next ecological trophic level.[172] While it is clear that energy and resources are required for autopoiesis and many biochemical processes have been identified, the exact set of mechanisms that enable life are not perfectly understood.[173]

All living organisms, like animals, plants, fungi, and bacteria, are composed of one or more cells. The cell is the smallest known unit of life. The cell has numerous subsystems for self-making components and accomplishing processes identified in Miller's living systems theory. Single-celled prokaryotic organisms like bacteria have a membrane to define the cell boundary and are filled with a mixture called cytoplasm along with subsystems to instruct cellular processes. Eukaryotic cells, which comprise plants and animals, have more delineated internal subsystems called organelles to accomplish the tasks of life.

Each of the organelles in eukaryotic cells, some listed in Figure 9-11, has specialized roles to enable the system to self-make. The nucleus holds DNA, the core depository of information. Ribosomes transcribe DNA into proteins, an essential building block of living systems, in places like the endoplasmic reticulum. From there the Golgi complex receives and sends packages of proteins. The mitochondria supports cellular respiration and making energy, the vacuoles store materials, and the lysosomes protect from bacterial invaders. These organelles, along with many other subsystems, work together to make a whole living system function.

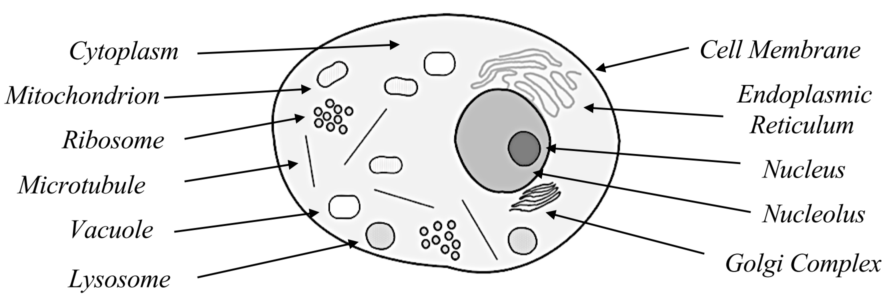


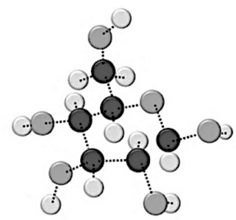
Figure 9-11 Animal Cell

Cells are composed of hydrogen, oxygen, nitrogen, carbon, and other trace elements, and these components form four categories of biomolecules: carbohydrates, nucleic acids, proteins, and lipids. Carbohydrates, like glucose, are long chains of carbon and hydrogen atoms that provide chemical energy. Nucleotides, such as the abbreviated *A, T, C* and *G* molecules, compose nucleic acids like DNA and RNA. Amino acids are molecules on a scale of a dozen or so atoms. Combinations of amino acids, on the order of several hundred, spiral and fold into proteins, which serve many functions. Lastly, lipids compose the cellular membrane and also support biochemical messaging. These molecular compounds are essential for life and is why carbohydrates, proteins, and lipids (fats) are all needed for balanced nutrition.

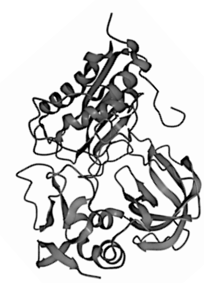
Cellular systems sustain organization through elegant and precise processes. One such process is the cell boundary, which is made of lipids. This membrane contains embedded proteins that act as miniature gates to regulate the flow of resources in and out of the cell. Inside the cell, carbohydrate sugars are used to generate energy to fuel processes. In the cell's nucleus, DNA is composed of millions of nucleotides. Special proteins transcribe genetic information from DNA to produce amino acid chains that fold into new proteins to carry out specific functions in the cell. Proteins serve a variety of functions, such as composing the cytoskeleton, a flexible matrix that supports the cell's structure and transports chemicals. These processes are related in a synergistic fashion to maintain a dynamic equilibrium, or homeostasis. Additionally, through intricate processes, the cell's DNA and other core organelles can reproduce and self-replicate.

Evidence supports the view that the biochemical building blocks of life can arise spontaneously within the conditions of an early Earth. The Miller-Urey experiment in 1953 demonstrated that amino acids and nucleotides can be created when adding electricity to common elements that would have been found on Earth prior to life.[174] Researchers have even discovered processes by which self-replicating RNA and cellular membranes could form within the conditions of early Earth.[175] These experiments demonstrate that highly ordered living systems can arise from natural processes. Additionally, life may not be such a random process within the bounds of cosmology. Scientists estimate that the Milky Way galaxy has an estimated 300 million or more habitable worlds that could sustain life.[176]

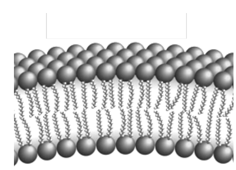
DNA
Nucleic Acids



Glucose
Carbohydrates



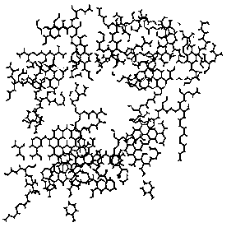
Protiens
Amino Acids



Cell Membrane
Lipids

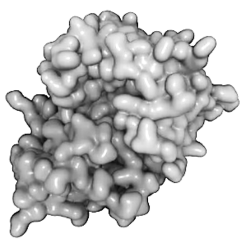
Figure 9-12 Molecular Building Blocks of Life

Electrochemical Fluids



Molecular Lattice

Folding Structure



Solvent Surface

Figure 9-13
Molecular Structure of Protein

Water provides an essential medium for the organization of living systems. No living thing on Earth is found absent of water.[177] The properties of water can be further understood by looking at the electron orbitals and the charges of water molecules. Individual water molecules form with a slight dipole, or a positive and negative end. Polar molecules and other molecules that dissolve easily in water are considered hydrophilic (Greek for "water-loving"). If a molecule is nonpolar, it repels water molecules and is called hydrophobic. The dynamic between hydrophobic molecules, such as lipids, and hydrophilic molecules, like glucose, plays an essential role in cells.

Water and polarity impact the interaction and formation of biomolecules, such as proteins. Proteins start as chains of amino acids, which then proceed to fold, twist, and spiral into their final shape. The protein folding process occurs in a water environment and is driven by the fact that hydrophobic amino acids will tend to fold toward the middle of a protein and away from the surrounding water. Proteins can be analyzed at different parallel levels, such as the underlying molecular lattice and the amino acid folding structures. The solvent surface defines the boundary that interfaces with water molecules, as shown in Figure 9-13.

Within the fluid medium of water, biochemicals interface like miniature magnetic puzzle pieces organized in a delicate mosaic. For example, the neurotransmitter serotonin has a specific shape and charge that fits with specialized receptors in the brain to trigger chemical cascades. There are many different types of reactions of biochemicals within a water medium, such as lock-and-key bonds between molecules, pumps to move specific molecules across concentration gradients, and enzymes that can speed up and enable chemical reactions. Receptors can even have resonant energy frequencies that match the molecules they interact with, which is suspected to aid in the ability to attract particular molecules.[178] These electromagnetic particles follow various processes over time in a fluid medium to create an organized living system.

The liquid crystal phase is another property of fluids that has important consequences for ordering and living systems. The liquid crystal phase is a partially ordered medium that occurs in temperature regions between a solid crystalline phase and fluid phase, shown in Figure 9-14. Liquid crystals thus straddle the line of maintaining rigid order and being open to change.

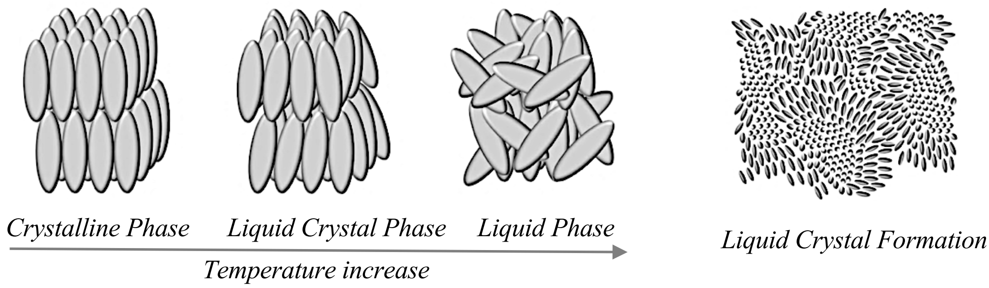


Figure 9-14 Liquid Crystals

Liquid crystals are common in biological systems. Spider silk, iridescent insect shells, and many other biopolymers assemble via liquid crystals processes.[179] Even cell membranes made of lipids feature a liquid crystal structure, as they are structurally connected yet malleable.[180] Many of the molecules within an organism, from membranes to other molecules on their surfaces, exist in a liquid crystal state. An organism as a whole most closely resembles a liquid crystal state, as cells and bodies are both solid, like matter, and malleable, like liquid.

Semi-ordered, liquid crystal arrangements can arise from the forces between the dipoles of water molecules, called hydrogen bonding. Hydrogen bonding gives water its surface tension, cohesion, and other properties. Hydrogen bonding can bring clusters of water molecules together to form a wide range of amorphous and semi-ordered clusters, like tetrahedral clusters shown in Figure 9-15.[181] Different cluster shapes can form around hydrophobic molecules, like proteins and cell membranes. Water clusters and liquid crystals can support the ordering of patterns, rather than random assortments.

Recent findings support the idea that much of the water structure along surfaces may not be randomly disordered, but instead exists in a liquid crystal phase. On the boundaries of hydrophilic surfaces water can create an "exclusion zone", where other structures suspended in water, like plastic microspheres, are repelled.[182] In the book *The 4th Phase of Water: Beyond Solid, Liquid and Vapor,* Professor Gerald Pollack theorizes that exclusion zones are created because water molecules are oriented into a liquid crystal phase. Many hydrophilic surfaces are created in biological systems and much of the water inside organisms may exist in exclusion zones as a type of liquid crystal, with specific pathways for diffusion, flow, and transportation. This would redefine the notion that water in cells exists as a disordered array of molecules, and points to greater order and structure.

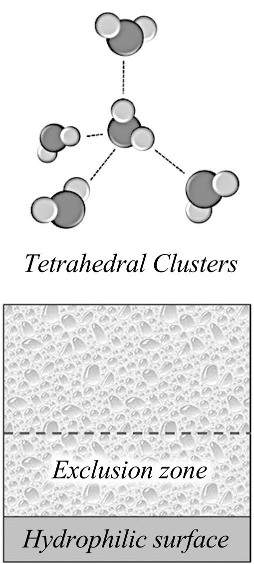


Figure 9-15 Water Clusters and Exclusion Zones

Morphology and Assembly

Morphology studies the forms of multicellular living systems and why particular structures emerge. While it is known that certain genes and biochemicals influence morphological forms, a full understanding of the biological mechanisms of growth remains elusive.[183] Modeling growth is exceptionally difficult, as biomolecular processes often contain nonlinear feedback loops and have high sensitivity to initial conditions.[184] Another factor is that morphology patterns, like the shape of a tree, are not fixed and uniquely adapt to environmental conditions, such as availability to water and light. Additionally, the process of evolution can change how organisms develop over time, which has led to many interesting structures like shells, nails, bones, and hair. A common thread in studying growth patterns is to think in terms of complex systems, self-organization, and adaptive evolution.

One organizing process in morphology is cell differentiation, which is a process where cells express a subset of their genetic code and become specialized, thereby reducing the possible states the cells can express. Pluripotent stem cells, or cells that exist in a pre-specialization state, differentiate into the ectoderm line (surface layer cells like skin), the mesoderm line (tissues and muscles), and the endoderm (inner layer organs). These different lines and examples of cells are summarized in Figure 9-16. Specialized cells can then pass genetic traits on to future cell generations as they replicate, which can create collections of similar cells that are organized together in groups. Similar cells tend to adhere to one another in what is called cell adhesion, and this creates ensembles of specialized cells that function together as organized subsystems.

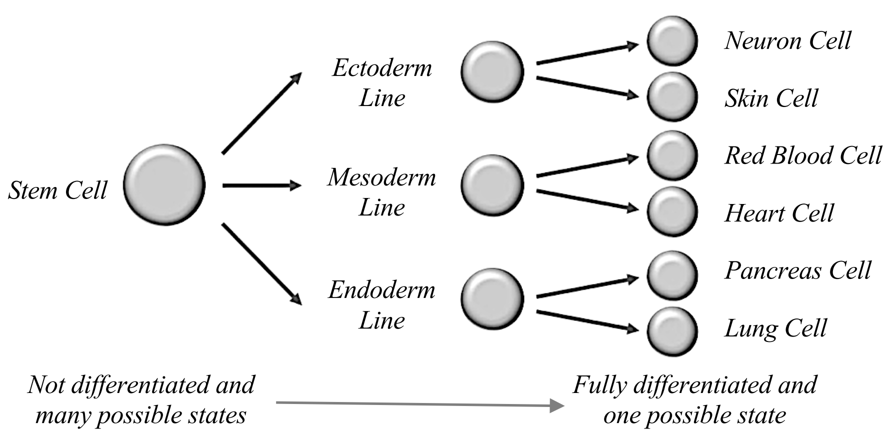


Figure 9-16 Stem Cell Differentiation

Electromagnetic fields play a critical role in biological self-assembly and organization. Inside and between cells, electrical potentials support various functions, such as the nervous system, brainwaves, and the pulse of the heart. While electromagnetic fields are typically understood only as a byproduct of chemical reactions, electromagnetic fields can also instruct morphological patterns. For example, coral reef growth can be accelerated in a given direction by exposure to an external electromagnetic field. Electromagnetic fields can influence the cell life cycle, cell proliferation, axon outgrowth, wound repair, and the establishment of left-right body asymmetry.[185] These are a few of many examples of the impact of electromagnetic fields on living systems and morphology patterns.

As mentioned in Chapter 8, a common morphological pattern in organisms is nested structures. For example, skeletal muscle is composed of a nested structure of smaller rope-like muscle fascicles. These muscle fascicles are subdivided into muscle fibers, as shown in Figure 9-17. Similar nested patterns occur in the circulatory and nervous systems, which have large pathways that contain many subdivisions. These fractal-like nested patterns can effectively distribute resources across many scales of size and be more error-tolerant to imperfections.[186] Nested morphology creates a cohesive structure from small to large scales that supports holistic organization.

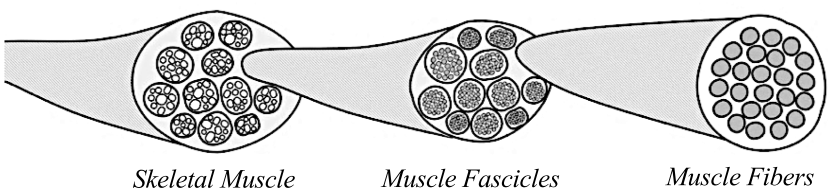


Figure 9-17 Nested Structures Muscle

At the anatomical level, living systems self-assemble to enable organized behavior. For example, skeletal bones and muscle tissue create a system of levers and pulleys to produce locomotion. While these results seem commonplace, functions like walking or swimming are highly ordered and would likely not form through the random arrangement of parts. This coordination of parts is much like a bicycle: the wheels, chain, frame, and other pieces need to be precisely oriented and interlinked to create the emergent property of locomotion. If the micro-level pieces of a bicycle were randomly connected, the macro-level result of motion would be nearly impossible to create. Biological morphological forms are similarly arranged in high levels of order for specialized functions.

Ecological Systems

Example 9.6 Trophic Cascades

Disparate aspects of an ecosystem can be interrelated. For example, introducing wolves to Yellowstone National Park influenced elk behavior, beaver populations, and vegetation, which changed how rivers flowed in the landscape.

Entire groups of organisms can be analyzed as living systems. Ecology studies the organizational patterns across many species over multiple generations. Different ecosystems around the world typically demonstrate similar organizing principles, such as the flow of energy, dynamic balance, cycles of change, complex networks, and nested systems. The interdisciplinary systems theorist Fritjof Capra proposed several common principles in ecosystems, summarized in Figure 9-18. Together, these principles set a foundation for studying ecosystems as complex networks that are open to the flow of energy and evolve over time.

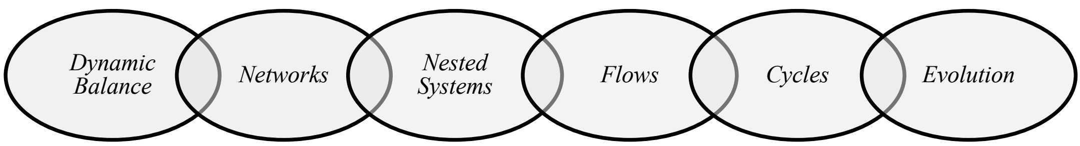


Figure 9-18 Organizing Factors in Ecology

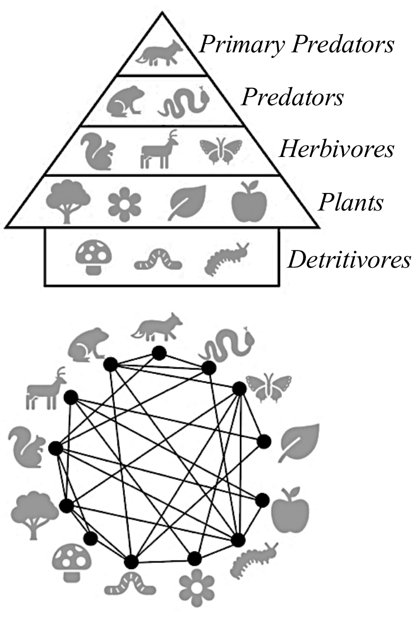


Figure 9-19 Food Webs

Food webs categorize different organisms in an ecosystem based on sources of energy. Food webs span plants (energy from the sun), to herbivores (energy from plants), to secondary predators (energy from herbivores), to primary predators (energy from secondary predators). While food webs are often pictured as a hierarchical pyramid or linear chain, they are better modeled as multilayered networks to represent the various types of interactions between organisms, as shown in Figure 9-19. Food web networks highlight the importance of connectivity and that a single species can influence many others.

Ecosystem food webs have a reciprocal cycle between life and death. When animals and plants die, detritivores like mushrooms and worms process the biomaterial which replenishes the soil with nutrients for future plant growth. This enables a sustainable and renewable system of energy exchange. The cycle of life and death also enables species to evolve over time and adapt to new environmental conditions.

A critically important organizing factor in ecosystems is that living systems evolve to suit their environment. The fossil record indicates that seemingly unrelated species like fish, birds, and mammals all share a common ancestry. Indeed, evidence supports that all organisms evolved from single-cell bacteria. Natural selection, posed by Charles Darwin in the mid-1800s, is the idea that less favorable genetic traits from random mutations have a lower chance of being passed on, causing evolution over many generations.

While natural selection is often imagined to be smoothly incremental, the fossil record reveals dramatic evolutionary leaps and extinctions in short time frames.[187] For example, the ability of cells to perform photosynthesis led to a rapid proliferation of new species. Another evolutionary jump occurred by the introduction of mitochondrial bacteria into animal cells, which provided the animal hosts with new functionalities for processing energy. Evolution can have dramatic changes over short periods, such as the mass extinction of dinosaurs, which is hypothesized to be caused by an asteroid impact and was followed by the rapid speciation of mammals. The evolution of living systems can be exposed to chaotic, volatile, and cataclysmic changes.

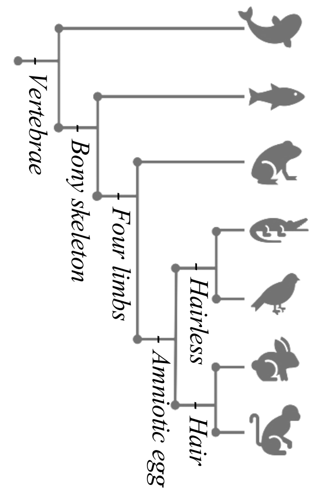


Figure 9-20
Vertebrae Evolution

Evolution has produced highly connected networks of interactions between organisms, such as competition and mutual support. In coral reefs, for example, many types of fish and corals live in proximity with various interactions, as depicted in Figure 9-21. Mutual support can be seen in the sea anemone and clownfish.[188] Sea anemones provide protection to clownfish as their tentacles only harm clownfish predators. At the same time, the clownfish cleans the anemone and wards off anemone predators. The vast web of ecological relations underscores the need to think in terms of connectivity, feedback, and emergence.

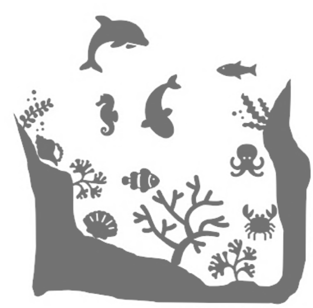


Figure 9-21
Diverse Ecosystems

Ecosystems are deeply related to geologic and atmospheric systems. For example, plants contribute to carbon dioxide fixation and alter the gas content within the Earth's atmosphere. Limestone rock is formed by the conglomeration of animal shells at the bottom of the sea. The abundant plant life in the Amazon rainforest generates moisture that influences rain cycles.[189] Over history, biology has become intertwined with Earth's processes including sedimentation, carbon exchange, oxygen exchange, and ocean salinity. Earth systems science and the so-called "Gaia theory" study how biology and the planet co-exist together as an interconnected system.

Intelligent Control

Intelligent systems can sense and adapt to the environment to create highly ordered states. Intelligent behavior is defined here to mean the ability for a given system to sense the environment to inform controlling specific outputs, decision-making, problem solving, or predictions of cause and effect. In this definition, synthetic systems like computers can perform intelligent procedures and intelligence is not limited to human cognition. In biological systems, intelligent behavior is exhibited in cells, organisms, and ecological groups. Intelligent systems can output particularly advantageous and rare states, which enables creating high levels of order in the environment.

Cybernetics is an interdisciplinary field that works to understand intelligent systems. The term cybernetics was coined by Norbert Wiener in 1948 to describe the scientific study of control and communication in the animal and the machine. [190] Cybernetic processes can interface with both engineering and biological systems, as well as broader psychological, sociological, and anthropological systems.

An integral concept in cybernetics is a control system, which senses the environment and uses a controller to determine specific outputs. A simple example is a microphone, control board, and speaker, as shown in Figure 9-22. The microphone senses sound in the environment and the controller determines the output of sound by a speaker. A cybernetic loop occurs when the control system's output is in feedback with the inputs, such as a microphone picking up a speaker. Another example is that some bacteria can sense toxins or favorable chemicals that control small hairs on the cell, called flagella, which move the bacterium to a more suitable location.[191] When the bacterium senses a favorable environment, it then stops. The cybernetic feedback of sensing and changing the environment is a critical process for how intelligent systems perform organized actions.

Example 9.7
Control System

A control system manages devices, like a thermostat controller turning on a heat in a room. While controllers can be modeled by their mechanistic forces, the system is goal seeking and resembles intrinsic purposes of teleology. A control system can have different control loops, that can increase order.

Closed-Loop Control:
Outputs can influence the inputs and sensor (e.g. thermostat)

Open-Loop Control:
Output doesn't influence the sensor (e.g. fixed timed)

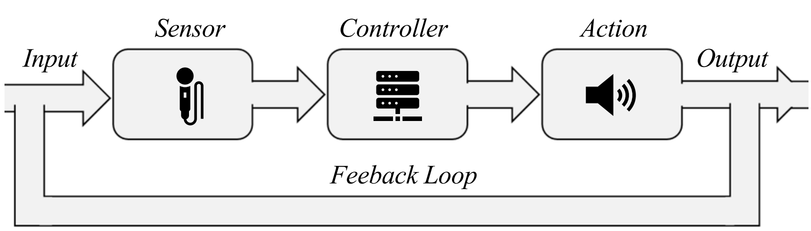


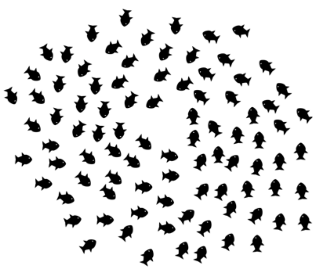
Figure 9-22 Cybernetic Loop

Cellular systems present a wide array of intelligent and organized behavior. Bacteria can perform a range of specialized actions, like creating tough cellular walls or changing their immunity to adapt to harsh conditions. More intelligent properties emerge when bacteria are together in a colony, such as collective decision making, adaptation, anticipation, and problem solving.[192] For example, groups of bacteria can send signals to turn genes on and off and even swap genes to promote antibiotic resistance. Researchers have even proposed that bacteria use chemical signaling to establish collective memory, group identity, and colony recognition.[193] These intelligent actions, and many others, allow bacteria to effectively self-organize, self-regulate, survive, and reproduce.

Plants are another prime example of intelligent behavior. Plants can sense environmental conditions like moisture, light, oxygen, gravity, sound, and chemical signals to accordingly adjust growth patterns.[194] Plants have photo-sensitive compounds that react to frequencies of light to determine growth and flowering cycles. Additionally, plants can sense the passage of time by using chemical processes that create circadian clocks. These sensory mechanisms enable plants to prepare for daily and seasonal cycles. Groups of plants can also perform intelligent and organized behaviors. For example, chemical tracers have allowed scientists to track nutrient exchange between the collections of trees in what is now understood to be an intricate underground communication signaling system mediated through fungal mycorrhizal networks.[195] This network of exchange and interdependence of plant and fungal life supports the survival and intelligent growth of the forest as a whole.

Animals have developed increasingly complex methods to sense the environment and act with intelligence. Through the nervous system and sensory organs, like the eye and ear, animals can sense the environment in ways not available to plants. Animals like fish, birds, and bees create detailed spatial memories to inform how to forage and migrate. Mammals, birds, and fish can even utilize tools to solve problems. For example, elephants can move boxes to stand on in order to solve the problem of retrieving out-of-reach bananas.[196] There are detailed modes of animal communication, like bird songs, bee dances, dolphin vocalizations, or sea lion barks, which convey messages for ordered behavior. Many animals, like the great apes, can recognize themselves in a mirror, as opposed to acting as if the reflection is another ape, which shows a high degree of self-awareness. These sensory abilities allow animals to analyze and order their surroundings in intelligent and controlled ways.

Herd of Land Mammals



School of Fish

Figure 9-23 Swarm Intelligence

> **Example 9.8** Self-organizing
>
> Self-organization occurs via:
>
> – *Strong dynamic nonlinearity, like feedback*
>
> – *Balance of expanding and consolidating forces*
>
> – *Multiple interactions*
>
> – *Availability of energy to overcome the tendancy for disorder to increase*

Groups of animals can organize through swarms to optimize collective activities. For example, land mammals can find efficient ways to travel long distances and forage for food by organizing into large herds. Fish often gather in large schools for food and protection. Swarm intelligence, which occurs as collective groups respond to environmental conditions, can aid problem solving. This can be seen in how groups of birds fly together as flocks to improve navigation and aerodynamic efficiency. Ant colonies self-organize in many ways and construct intricate underground tunnels. Swarm intelligence is one means for systems to self-organize and effectively address problems.

Humans have developed particularly high levels of intelligence, order, and self-organization. The complex physical and informational interactions of humans can be studied through the emergent models of anthropology and social sciences, which are comprised of systems of social agents as well as associations to abstract ideas. Humans have collectively organized into large groups and create economic systems of exchange. Humanities' abstract communication tools, like writing, has supported activities like building infrastructure and creating political systems. Language, which refers to abstract concepts via an open-ended system of symbols, plays a prevalent role in human intelligence and allows a practically limitless medium to express concepts and cultural ideas. Humans have developed storytelling, written histories, and scientific knowledge to pass down informational artifacts over time.

In humans, cognition primarily occurs in the central nervous system (brain and spinal cord), which processes sensory data and memory to direct action. Sensory organs serve as the mechanism to receive inputs. These inputs then pass through the cognitive system, which informs future decisions from past experiences. After processing information, biological motor functions enable the output of changes to the environment, as graphically displayed in Figure 9-24. This sequence is similar to cybernetic feedback, with inputs, a controller, and outputs. Cognitive processes give humans the ability to design highly ordered energetic and informational states in the world.

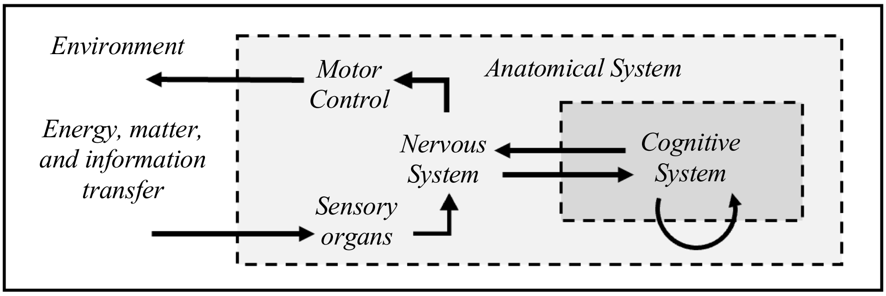


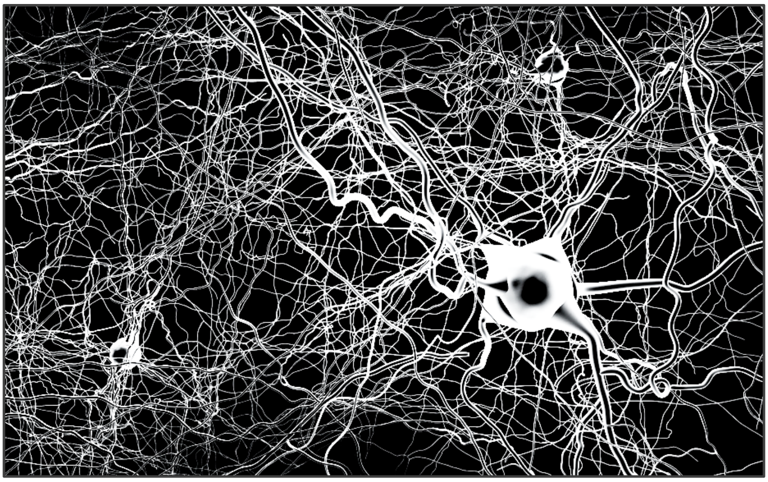
Figure 9-24 Cognitive Organization and the Environment

Cognitive processing is more complex than a fixed cybernetic controlling system because the decision-making device, or brain, is under continuous change. This means that the same sensory input can produce different outputs at different times, due to the controller (brain) reformulating the decision-making rules. The brain is continuously processing information through the stages of sensory memory, short-term memory, long-term memory, unconscious memory, as well as forgetting information, to create efficient models of the world. Cognitive processes allow humans to make sense of the environment and adaptively design ordered systems in a changing world.

Summary

Ordering is a property involved in many complex systems and plays a critical role in the emergence of life. Biological systems create highly ordered states through energy-harnessing reactions and other novel processes that bend the laws of chemistry to fantastic extremes. While abiding by the laws of physics and entropy, life is able to use energy and resources in an open exchange with the environment to create patterns that self-organize and self-reproduce. Collections of plants and animals form ecosystems that create new emergent patterns and evolutionary trends. Intelligent control systems are able to sense the environment and effectively regulate and govern a wide range of ordered activity. Living systems use control and communication to intelligently sense and react to the environment, evolve into more advantageous patterns, and even create socioeconomical systems capable of generating knowledge of nature. Ordering is a critical concept to grasp how complex open systems, such as life and control systems, organize the cosmos.

Chapter 10 Information



Example 10.1
Neural Network

The brain is formed with trillions of connected neurons and can process information as a single cohesive system.

Information arises when the states of material objects are used to mark and process symbolic messages. This can be seen in DNA sequences, neural networks, or computers that transmit, store, and process informational states. Information theory is at the frontier of many disciplines studying complex systems, from quantum physics to computer theory to cognitive science. Many informational systems function with high levels of connectivity, interdependency, and emergent properties, which reinforces the need to analyze nature through a systems-based worldview.

Information relates to the quantity of unexpected results, or surprisal probability, that is addressed by a piece of data. If, for example, an event has a 100% chance of occurring, then no information will be provided by knowing that the event occurred. However, if it has a 50% chance of occurring, then when the event occurs it would provide information, and specifically one bit of information. Shannon entropy $H(X)$ quantifies the average expected information over many variables. An event with a lot of information will have a high Shannon entropy, while an event with less information will have lower Shannon entropy. A system is considered to possess information when the Shannon entropy is greater than zero, as shown in Figure 10-1.

$$S : \{H(X) > 0\}$$

Figure 10-1 Equation for Information

Example 10.2 Surprisal and Information

An outcome's message that is surprising has more information. An event with 50% chance of occurring (*A* or *B*) conveys one bit of information. An event with that has a 25% chance (*A, B, C,* or *D)* conveys 2 bits. An event with 100% chance of occurring (*A* or *A*) has no information.

Equal Choices	$H(X)$
(A)	*0 Bit*
(A, B)	*1 Bit*
(A, B, C, D)	*2 Bit*

Informational States

Claude Shannon introduced the formal study of information in the 1948 paper *Mathematical Theory of Communication.*[197] This paper introduced the unit of a bit, which can measure the quantity of information. Information theory provided the foundation for modern communication and computational systems, including cell phones and the Internet.

Shannon's essential insight was to quantify information based on how surprising a given event and message is expected to be. The information content, or self-information $I(x)$, was chosen to meet the axioms that: an event with a 100% probability yields no information, the probability of an event is inversely correlated to the information it yields, and the information of two independent events together equals the sum of the information of each individual event. The formula $I(x) = log(1/p_x)$ follows these axioms, as the logarithm allows independent probabilities to linearly sum together. When each state is equally probable, such as rolling a uniform die with equal chances for every side, the information is proportional to the logarithm of the number of states, $I(x) = log(states)$. The unit of the bit uses the base 2 logarithm to quantify information, following $log_2(1/p_x)$ or $log_2(states)$, though other bases can be chosen.

Shannon entropy is the average expected information equal to $\Sigma p_x log(1/p_x) = p_1 log(1/p_1) + p_2 log(1/p_2) + \ldots$. For example, consider a symbol that has an equal chance of being one of four choices. When the symbol is chosen from the letters *(A, A, A, A)* there is a 100% chance of an outcome of *A*, so no information is provided by the selection event. When chosen from *(A, A, B, B)* there is a 50% chance for an outcome of A and a 50% chance for B, creating 1 bit of Shannon entropy. When choosing from *(A, A, A, B)*, there is a 75% chance for an outcome of *A* and a 25% chance for *B*, creating 0.81 bits of information, as shown in Figure 10-2. On average, the third scenario provides less information than the second, because *A* occurs ¾ of the time, rather than ½ of the time.

Example 10.3
Shannon Entropy

The information $I(x)$ of an event x is the log of one divided by the event's probability p_x:

$$I(x) = log(1/p_x)$$

Shannon entropy $H(x)$ is the average amount of information for multiple events:

$$H(X) = \Sigma p_x log(1/p_x)$$

Shannon entropy has a non-equal, and often opposite, relation to thermodynamic entropy. Shannon entropy is the bits of data provide by a message at any level, while thermodynamic entropy is the missing information about the energetic microstates.

$$H(X) \neq Entropy_{Thermo}$$

Equal Choices	$H(X)$	$= \Sigma p_x log_2(1/p_x)$
(A, A, A, A)	*0 Bits*	$= (1)log_2(1)$
(A, A, B, B)	*1 Bits*	$= (1/2)log_2(2) + (1/2)log_2(2)$
(A, A, A, B)	*0.81 Bits*	$= (3/4)log_2(4/3) + (1/4)log_2(4)$

Figure 10-2 Shannon Entropy

Matter and energy (e.g. a written book, a strand of DNA, or a radio wave) can serve as markers of information and be encoded and decoded into symbolic messages. The Shannon-Weaver model, shown in Figure 10-3, studies communication systems of transmitters, which encode the messages to be sent over a channel, and receivers, which decode the message. Noise can also degrade information as the message travels over the channel. In a telephone call, for example, sound signals are encoded, sent over a radio wave channel that can be exposed to noise, and decoded for the receiver. Communication channels can be very efficient when the marker of information is easy to send, such as the radio waves of telephone calls. However, markers of information are limited to properties of matter and energy, which tend to increase in entropy and degrade over time, leading to noise.

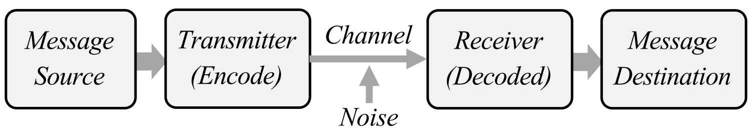


Figure 10-3 Shannon-Weaver Communication Model

Mutual information measures the information shared between the sender (x_{Input}) and recipient (y_{Output}) over a channel. If x and y are independent, then observing x does not give any information about y, and the mutual information is zero. However, if x fully determines y, then the information of y is conveyed by x. The mutual information of x and y, written $I(x,y)$, also equals the Shannon entropy of x minus the conditional entropy $H(x|y)$, which is the amount of information needed to describe x with y known. Additionally, noise can increase entropy and over a channel following $y_{Output} = x_{Input} + noise$. Shannon's noisy-channel coding theorem establishes the rate at which communication can occur with minimal errors. It is critical that physical markers provide more symbolic information than background noise, which requires open energetic systems to counter entropy and noise.

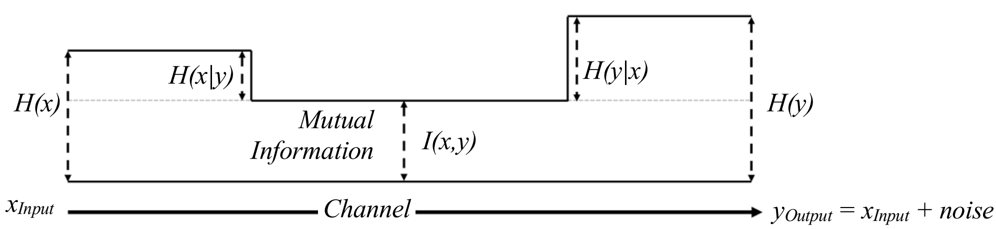


Figure 10-4 Mutual Information

Quantum Information

Information plays an essential role in quantum systems. Quantum particles are modeled through wave packets of probability, such as a Gaussian distribution where the probability decreases farther away from the average. The Schrödinger equation describes how the quantum wavefunction Ψ evolves over time and is derived by applying the principle of least action to a quantum field. To calculate polarization and intrinsic spin, quantum waves are described by both real and imaginary ($i^2 = -1$) components. Following Born's rule, a key postulate of quantum mechanics, the probability of finding a particle at a given region is proportional to the square of the wavefunction Ψ^2, which removes the imaginary component and leaves only a real component. In an observed interaction, quantum probability fields collapse to define the position and momentum of particles within a certainty range.

> **Example 10.4**
> Schrödinger Equation
>
> The Schrödinger equation models how the quantum wave function Ψ changes over time from the system's Hamiltonian $\boldsymbol{H}$, a measure of the total energy, and Planck's constant $\hbar$.
>
> $\boldsymbol{H}(\Psi) = i\hbar\partial\Psi / \partial t$
>
> The equation describes one singular field, that can have many wave packets and also allow for entanglement.

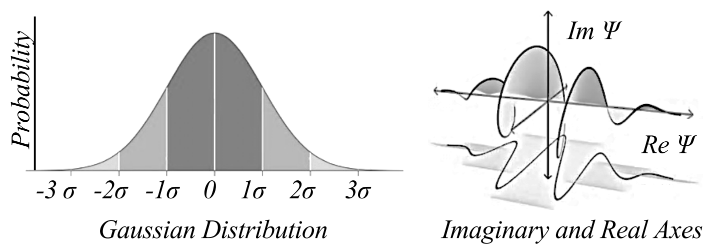


Figure 10-5 Quantum Probability Field

Quantum theory questions the premise that matter has a well-defined location in isolation. Two common interpretations of quantum theory are that particles do not have a well-defined location when not interacting (the Copenhagen interpretation) or that particles exist in well-defined locations when not interacting, but are influenced by hidden variables (hidden variable theory). However, Bell's 1964 theorem showed that quantum theory can make predictions which would be violated if local hidden variables existed.[198] A hidden variable solution to quantum mechanics could be non-local, meaning particles can be influenced instantaneously across distances, but no theories have so far developed greater predictive power.[199] Quantum physics supports the view that particles either do not have a position when not interacting, or they are governed by non-local variables. Both options present a radical shift from classical physics, where particles and fields have well-defined values at each point in time and are influence by local forces.

Another view of quantum theory is the information interpretation, where the primary building blocks of the universe are information interactions rather than local, isolated, objects. In these models, quantum measurement events occur as information waves interact. Probability fields can be defined, within uncertainty limits, through informational interactions with other probability fields. Quantum information theory contrasts the Newtonian matter-based view where particles have defined locations and exist when not being measured, as summarized in Figure 10-6. Quantum mechanics supports the systems thinking concept that identity is acquired via interdependent relations, because interactions are required to collapse a quantum probability field to an outcome.

Example 10.5 Relational Quantum Interpretation

Relational quantum mechanics is the view that physical variables only take on values through interactions between multiple systems.

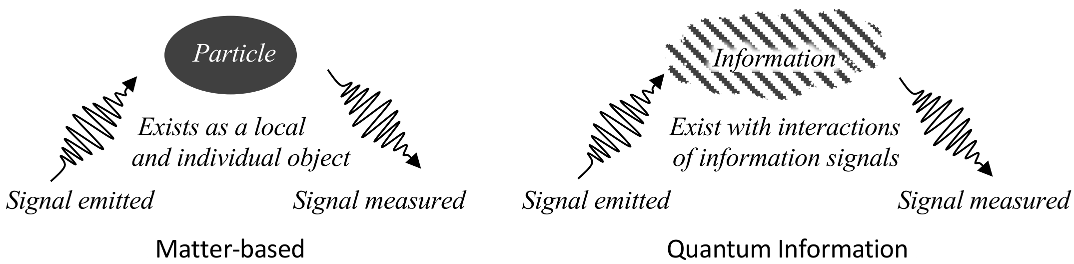


Figure 10-6 Matter-based vs. Quantum Information Interpretations

Another unexpected behavior of quantum systems is instant action at a distance. Distant quantum particles can affect one another through entanglement. For example, after two coupled photons scatter apart, measuring one photon state will instantly provide information about the other photon's state due to conservation laws. Entanglement is not just theoretical and has been experimentally demonstrated.[200] Due to its non-local nature, entanglement reframes the traditional view that particles are acted upon only by neighboring forces and fields. Quantum theory points to a view of nature that is entangled, non-local, and inherently probabilistic.

Quantum information—measured through von Neumann entropy—is posed to be conserved, which seemingly contradicts the increase of thermodynamic entropy. These ideas can indeed coexist because in thermodynamic entropy refers to how much information is missing prior to measurement, while in quantum theory, von Neumann entropy relates to the states present in measurement. Interacting quantum states can become entangled, which reduces the number of measurable outcomes. This allows quantum information to be conserved while thermodynamic entropy increases.

Example 10.6 Von Nuemann Entropy

Von Neumann entropy E_{VN} analyzes the uncertainty of quantum probability density matrices p using the Trace (tr), which is a type of matrix summation:

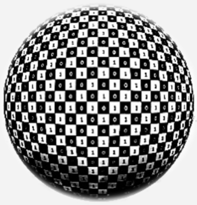
$$E_{VN} = tr(p_x \ln(1/p_x))$$

Von Neumann entropy measures the total measurable information in a quantum system—which is conserved.

Holographic Equivalence

Example 10.7
Black Hole Entropy

A black hole's entropy is proportional to the surface area of the event horizon. This result suggests that the information and entropy of the entire volume is encoded on the boundary, similar to how a holographic surface encodes an entire volume.



Information plays a role in modeling spacetime, especially black holes. Black holes are objects with enough mass to create an event horizon, a boundary where gravity is so strong it traps light. Similar to energy conservation, information is posed to be conserved in quantum physics.[201] However, black holes create an information paradox because the event horizon prevents signals from escaping, effectively destroying information that enters. To add to the puzzle, Stephen Hawking showed that black holes absorb particle-antiparticle pairs that arise from quantum uncertainties on the event horizon.[202] This effect, known as Hawking radiation, means that black holes radiate energy and evaporate, seemingly losing the internal information. A suspected solution to this information paradox is the holographic equivalence.[203] The holographic equivalence allows information of a volume to be encoded on boundary surfaces and to conserve information in black holes.

A hologram has a number of interesting properties. One property is that a 2-D holographic plate, or surface, can encode the information of an entire 3-D volume. Additionally, each part of a 2-D holographic plate has the information of the entire 3-D hologram. Even a small section of the holographic plate will retain an image of the entire 3-D hologram, but will be a lower resolution, as shown in Figure 10-7. This property is similar to how each piece of a fractal can reflect the whole pattern. Holographic information also displays symmetries that relate to entangled quantum systems.[204] In both holograms and entanglement, the whole system relates through non-local relations. Changing the underlying projection pattern can instantly change each sub-division.

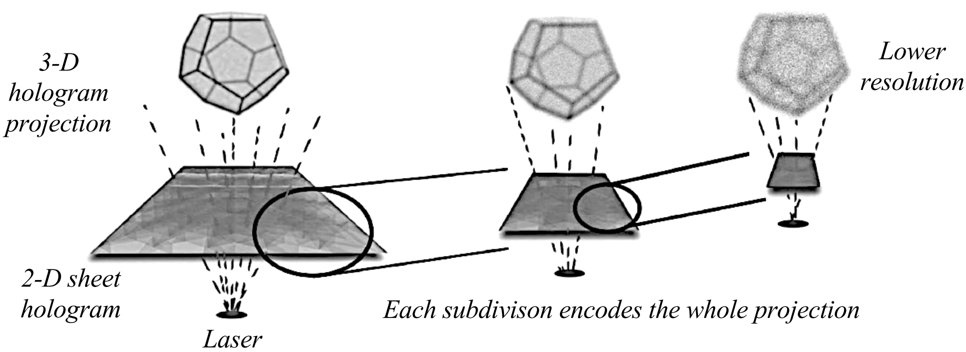


Figure 10-7 Holographic Information Encoding

The holographic, or Ads/CFT, equivalence introduces new ideas of how information is encoded in space. The correspondence allows a version of string theory in a type of spacetime, called Anti-de Sitter space (Ads), to be equally represented as a holographic boundary following a conformal field theory (CFT). Following this idea, all the information within a given space can be equally represented as a holographic encoding of a boundary surface. This solves the black hole information paradox because matter on the inside of a black hole can be conserved on the boundary and later released via Hawking radiation, while never being truly destroyed. Extending this idea further, all the information within the universe may be encodable on the cosmological boundary.[205] Anti-de Sitter space (Ads), the space inside holographic boundaries, is locally equivalent to 4-D Minkowski spacetime (described in Chapter 5) and makes observations compatible with accepted cosmology.[206] The holographic equivalence proposes a new symmetry, where volumetric information can be equally mapped to holographic boundaries.

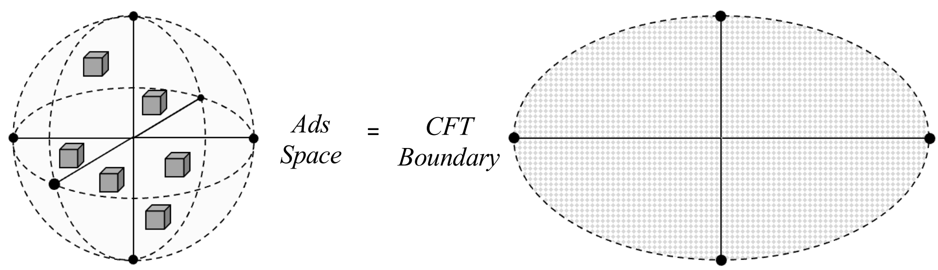


Figure 10-8 Holographic Universe

Black hole boundaries and the holographic principle has implications for the maximum amount of information that can be stored within a bounded space. The Bekenstein bound states that the maximum entropy is proportional to the surface area and energy. The amount of Shannon entropy, which relates to macro-level degrees of freedom (e.g. transistor states) could get closer to, but never surpass the thermodynamic entropy, which relates to the degrees of freedom of the underlying energy. The Bekenstein bound sets a limit to the maximum amount of information that can be contained in a space or be required to describe any energetic microstates contained. The Bekenstein bound has very large values. For example, containing the energy of 1 kilogram (following $E = mc^2$) in a sphere with a radius of 1 centimeter creates a maximum entropy of $\sim 10^{52}$ bits. This is truly a mind-boggling amount of data, and 2020 estimates of world digital data storage is 27 orders of magnitude less at $\sim 10^{25}$ bits.

Computational Systems

Computational systems physically mark and manipulate information to follow logical procedures. In 1938, Claude Shannon showed that electrotonic switch arrangements can represent logical procedures and Boolean algebra, where *1* is true and *0* is false.[207] Networks of circuits can be used to perform logical operations (*and*, *or*, *implies*, ect.) as well as store memory. Modern computers receive inputs of information via keyboards and other sensors, use circuit boards to perform logical processes, and output the results of those processes to screens and other mediums. By processing on-off states (*1* or *0*) of these transistors, computers can perform many complicated actions like solving math equations, analyzing languages, and supporting communication. Computers have revolutionized society and increased the ease of processing and sharing information.

Processing information is an emergent behavior that arises in a subset of physical systems. For example, a mess of wires cannot compute non-trivial information unless properly arranged. If properly arranged, a physical circuit can be equivalently mapped to an emergent computer model of Boolean algebra. In its general form, information and computation are substrate-independent, and can be equivalently represented on any physical system capable of marking and manipulating messages such as electronic circus, floppy disks, flash drives, beads of a rope, and DNA strands. However, the processing of information in living systems is related to the substrate, because following autopoiesis the information procedures (software) must self-make the physical system (hardware).

Informational systems can widely vary in data storage density, speed of reading or writing data, and power usage. Examples of these metrics for hard disks, flash drives, and DNA are shown in Figure 10-9. While flash drives are more efficient than hard disks, neither can compare to the density or power efficiency of DNA.[208] Optimizing the speed, density, and power to process data are critical factors in creating information systems, in everything from computers to cells.

Information Medium:	*Hard Disk*	*Flash Drive*	*DNA*
Read-Write Speed: *(bits / second)*	*250*	*10,000*	*>10,000*
Data Density: *(bits / cm^3)*	10^{13}	10^{16}	10^{19}
Power Efficiency: *(gigabytes / watt)*	*25*	*50*	$>10^{10}$
Storage Lifetime: *(years)*	*>10*	*>10*	*>100*

Figure 10-9 Table of Information Mediums

Since the early 1980s, engineers have explored building quantum computers, which exploit the properties of superimposed and entangled quantum systems. While a bit can exist in two states, *0* or *1*, a quantum bit, or qubit, can exist in a superposition of possible *0* to *1* states depending on the other qubits. A classical bit will only depend on its own value and a function that translates the bit to a *0* or *1* value only depends on the value of the bit. In contrast, the value of a qubit depends on the values of the other qubits, rather than being fixed and independent. This means the function to find the value of one qubit can include all the other entangled qubits, as shown in Figure 10-10. Due to the vast number of superimposed states and novel properties like parallel processing, quantum computers can use new algorithms to solve problems not available to classical computers.

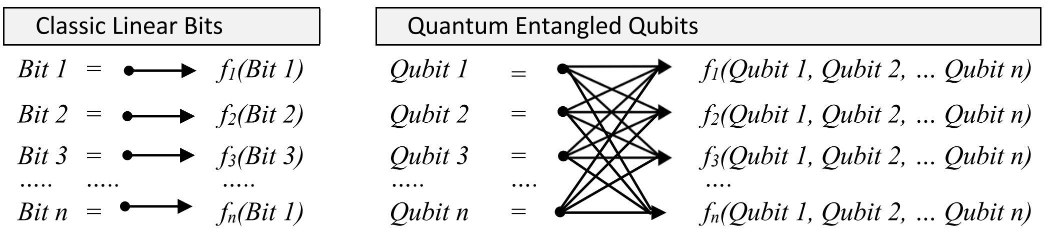


Figure 10-10 Quantum Qubits

Information systems have a wide spectrum of data units. In a binary computer, each *n* bit has two states (*0* or *1*) that create 2^n total states. In DNA, each *n* gene has four possible states of *A, T, C,* and *G* that create 4^n total possibilities. Additionally, three DNA states couple to create 64 units (4^3), called codons, that instruct which amino acids are used to build proteins. In the brain, each neuron has approximately 7,000 synaptic connections.[209] This means the number of connection states between *n* neurons scales to ~7000^n, generating extraordinarily high results. In a quantum computer, each qubit's state is defined over a fully entangled probabilistic wavefunction Ψ.

Information System	*Computer*	*DNA*	*Brain*	*Quantum CPU*
Data units	*Bit*	*Nucleotides*	*Neuron*	*Qubits*
States per unit	*0, 1*	*A, C, G, T*	*~7,000*	$\Psi = f(n)$
States of n units	2^n	4^n	~7000^n	2^n
Interaction of units	*Linear*	*Coupled*	*Network*	*Entanglement*

Figure 10-11 Computing Units of Different Information Systems

Computational Complexity

Example 10.8
Time Complexity

Time complexity refers to the number of steps, or time *t,* to solve an algorithim with an input size *n*. Time complexity classes include:

Linear: The time to solve scales at the same rate as the input, $t = cn$, where *c* is a constant. These problems are very fast to solve.

Polynomial (P): The time scales faster than the size of the input, $t = n^c$, but is efficient to solve.

Nonpolynomial (NP): The time scales exponentially, following $t = 2^{n^\wedge c}$. These algorithms are inefficient, and are not faster than brute-force testing all possible solutions.

One measure of algorithmic information is Kolmogorov complexity, which refers to the minimum set of symbols needed to transmit a message without losing detail.[210] For example, the 20-character string [*xyxyxyxyxyxyxyxyxyxy*] can be compressed to the 7-charater string with instructions to print *"p"* the string (*xy)* multiple times [*p(xy)10]*. By contrast, the 20-character string [*vdfhfqstonwogjwetbqg*] cannot be compressed and has greater Kolmogorov complexity. An example of data compression occurs when making ZIP files in a computer, which works to reduce the total bits of data, and lower the Kolmogorov complexity, for a file.

Intricate patterns can have surprisingly low Kolmogorov complexity. For example, the irrational number $\sqrt{2}$ creates an endless and non-repeating sequence of digits *1.414213…*, but this sequence can be reduced to a short algorithm to calculate digits with increasing precision. Similarly, fractal patterns have efficient data compression sequences because rules are repeated to produce intricate details from small to large scales. Many systems utilize efficient sequences to form seemingly complex patterns with low amounts of data resources.

Informational systems often employ clever sequences to reduce algorithm processes. For example, DNA sequences do not delineate every cellular position and the anatomy in the organism, but rather provide basic rules for self-organizing.[211] Similarly, the brain has various shortcuts, or heuristics, for common problems it needs to solve. These data-efficient strategies lower memory use.

The required steps, or time, to complete a computation is another complexity measure. One classification of these problems is $P = NP$ or $P \neq NP$, which asks whether every problem for which a solution can be checked quickly in polynomial (*P)* time can be solved in polynomial time, or if an inefficient nonpolynomial *(NP)* equation is required. The generation of even numbers is $P = NP$, meaning the results are both quickly checkable (divided by 2) and quick to generate (any number doubled). On the other hand, the generation of prime numbers is believed to be $P \neq NP$, as there is no quick calculation to find new prime numbers of high value, even though there is a time efficient algorithm to check if a number is prime (divisible only by one and itself). Another $P \neq NP$ problem occurs in the game of Sudoku, a puzzle of digits placed in a 2-D grid following a set of constraints. A winning Sudoku solution can be efficiently checked, but solutions are difficult to generate.[212]

A computational problem is reducible when there is an algorithm to transform it into another solved problem. For example, if problem *A* is unknown, but can be transformed into problem *B* with a known solution, then problem *A* can be solved. Turing reducibility takes this further and is the ability to reduce any given problem to a single yes or no output after repeating a finite number of computation steps. Some problems have a decidable yes or no answer, while other problems, like the halting problem, are irreducible with no finite algorithm and undecidable to a final yes or no solution.

Computation irreducibility, a term coined by Stephen Wolfram in *A New Kind of Science,* distinguishes systems that cannot be reduced with efficient algorithms. Many models of complex natural systems, such as nonlinear networks, fluid turbulence, and protein folding, cannot be solved via a finite number of steps and are not Turing reducible. Traditional science tends to focus on systems that can be reduced by efficient algorithms and have linear components. Wolfram argues that science should explore complex, nonlinear, and irreducible patterns.

Computation irreducibility occurs in uncomputable numbers, which cannot be solved in an efficient manner. Computable numbers, like Euler's number *(e = 2.718…)* and pi *(π = 3.1415…)*, can be solved with efficient algorithms. Increasingly precise results can be found by repeating rules, even if they are irrational with an infinite number of non-repeating digits. In contrast, Chaitin's constant—the sum of the probabilities that a halting program will stop—and the "busy beaver function"—which finds the largest output of a given halting program—are uncomputable. These uncomputable numbers do not become increasingly precise over incremental steps.

Computable Algorithms	*Euler's Number (e)*	$= \sum_{n=0}^{\infty} \frac{1}{n!}$	$= \frac{1}{1} + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots$
	Pi (π)	$= \sum_{n=0}^{\infty} \frac{n! 2^{n+1}}{(2n+1)!}$	$= 1 \cdot 2 + \frac{1 \cdot 2^3}{3 \cdot 5} + \frac{1 \cdot 2^4 \cdot 3}{3 \cdot 5 \cdot 7} + \frac{1 \cdot 2^5 \cdot 3 \cdot 4}{3 \cdot 5 \cdot 7 \cdot 9} + \ldots$
Uncomputable Algorithms	*Chaitin's constant (Ω)*	$= \sum_{p \text{ halts}} 2^{-\lvert p \rvert}$	$\lvert p \rvert$ *= size of bits of program that halts*
	Busy beaver (BB)	$= \sum_{n=1}^{\infty} 2^{-BB(n)}$	*BB(n) = largest halting program of size n*

Figure 10-12 Computable vs. Uncomputable Numbers

Artificial Neural Networks

Information can be manipulated in useful ways with artificial neural networks. Artificial neural networks are loosely inspired by biological neurons and process information through a network of nodes connected by specific rules. Much like the brain's neurons, artificial neural networks can solve complex problems, like pattern recognition, text generation, image creation, and even improve accuracy over time. Artificial neural networks are used in artificial intelligence (A.I.) software to solve complex problems across a wide range of fields.

Artificial neural networks are composed of multiple nodes that have specific rules to transform input data to output results. The nodes in neural networks can reference one another, represented by arrows, to solve problems, like image recognition. For example, consider an image of two pixels that can either be black or white and the desired outcome is to determine if the image has a solid or checkered pattern. To do this, the first layer of nodes senses if each pixel is black or white. From there, each of the nodes on the second layer references two of the previous nodes, represented by lines, to determine if the input image is solid black, solid white, or one of two checkered patterns. Finally, the output nodes reference the results from the previous layer to determine if the pattern falls into a solid output or checkered output category. The checkered example in Figure 10-13 follows the nodes' rules from input to output result.

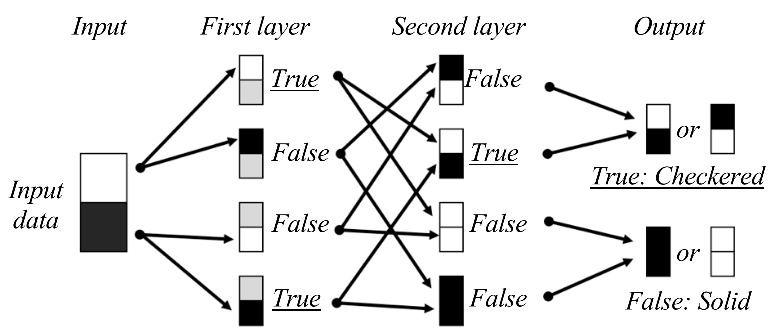


Figure 10-13 Simplified Neural Network

To take a more complicated example, consider the task of identifying a flower from an image that is split into a grid of pixels. The ability for a human to enter the rules of identifying flowers is extremely difficult because there is so much variety in flowers. A more practical way to take on this task is to start with random rules that each node follows, which will produce many errors. After a series of tests where an outside source confirms if the answer is correct or not, the next step is to change the rules of the nodes in a direction that will minimize the error. This process is called back propagation and is a way for neural networks to "learn" and improve over time, pictured in Figure 10-14. After many testing phases, the neural network rules will become better at identifying flowers in a photo. Mathematically, this process uses vector calculus and follows the gradient to change the node rules in a direction that minimizes error.[213] Neural networks show how an information system can develop and adapt a set of rules to effectively process complicated information.

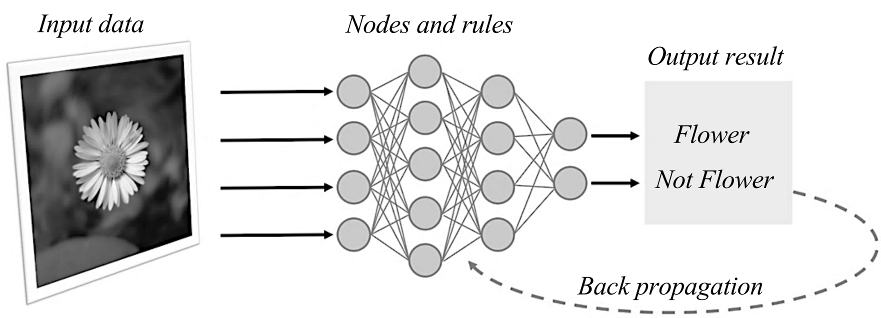


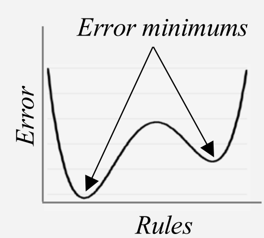
Figure 10-14 Neural Network Learning and Back Propagation

The neural network rules created from back propagation can be non-intuitive, messy, and illogical, contrasting the example of identifying patterns in four pixels. However, the seemingly random rules in neural networks can generate extremely effective results with proper test data. Neural networks also represent a universal function, in that they can approximate any possible function, like text to text, text to image, image to text, and even be used to predict results of complex physics problems like some protein folding geometries. However, artificial neural networks typically rely on inputting known test data, do not often provide insights into the underlying dynamics, and do not analytically solve problems in compact forms like physics equations.

An interesting attribute of neural networks is that two sets of neural networks can have completely different rules, or ways of deciphering reality, but end up with similar predictive power. Even though neural networks are much simpler than the human brain, applying this concept to the human neural network could mean that two or more completely different mental models of reality may lead to similar accuracies in real-world testing.

Example 10.9 Reducing Neural Network Errors

The gradient, or slope, is followed to change rules to reduce neural network errors. Also, multiple rules may minimize errors.



Perception and Memory

One information process common in living systems is spatial perception. Animals can create perceptual models of shapes, colors, and trajectories in space, which are essential for locomotion, migration, and identifying threats or opportunities. [214] The basic process involves input signals from sensory organs that are processed in the brain to create perceptual representations, as shown in Figure 10-15. Perception in animals has high levels of information feedback as organisms can influence the environment and subsequently change the sensory signals received. While many brain regions and processes related to vision have been identified, such as the visual cortex in the back of the head, the exact mechanism of perceptual modeling is still a topic of ongoing research and not completely understood.[215]

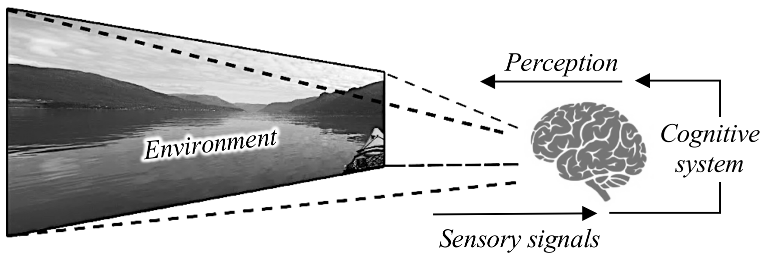


Figure 10-15 Cognitive Modeling of Environment

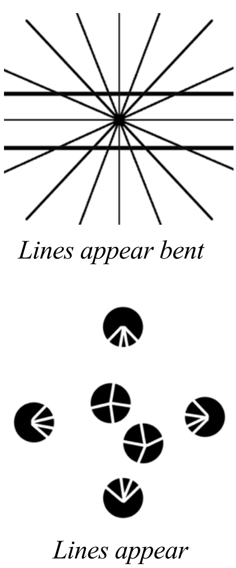


Figure 10-16
Visual Illusions

While it may seem that perceptions of the environment should match reality, this is not always the case. For example, visual illusions show that sensory data can be grossly distorted. One common illusion is that a horizontal line will appear to be curved when it crosses other lines converging to a center point. Another visual illusion is that the brain will project lines onto a pattern when just the tail ends of the lines are provided, as shown in Figure 10-16. Visual illusions show that the brain can alter, take away, or add information when perceiving the environment. Cognitive representations often do this to save energy and memory usage when observing patterns. More generally, the brain perceives reality as what is most evolutionarily suitable, functional, and energy efficient, not necessarily what is true.[216]

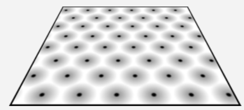
A startling property of spatial representation is the brain's ability to efficiently memorize vast amounts of information, which is accomplished through an elegant method of neural connectivity. When an animal is in a particular region in space, such as a mouse walking over the same point in a maze, neurons called place cells occasionally fire. In addition, the brain also contains grid cell neurons that fire when an animal passes regular interval points in a grid-like map. A connected network of grid cells can identify specific locations with extreme efficiency, as explained more in Example 10.10. For example, 1000 place cells can only represent 10^3 individual locations, but 1000 grid cells can hypothetically represent ~10^{30} locations. Animals utilize both place and grid cells to efficiently memorize and model the spatial environment.

An intriguing theory of how the brain efficiently models and memorizes objects is hierarchical temporal memory (HTM). Mathematically, HTM is an artificial neural network model where each node has thousands of connections to other neurons (simulating both synapse and dendrites) and learns unlabeled data in an unsupervised way by creating new synapses.[217] Following HTM theory, representations are constructed through recursive hierarchies of rearrangeable units. For example, when the object of a car is thought of, it is perceived as a unit where the subcomponents or combinations apply to other units. For example, the unit of a car can be combined with a store to think about a store that sells cars. The unit of the store can also be associated with other units. A car also contains many sub-elements, like an engine, which is a unit that can simultaneously apply to other items that contain an engine, like a boat. HTM theory poses that the brain can efficiently store memories by flexibly arranging elements in subsets or supersets, compared to each item having its own identity.

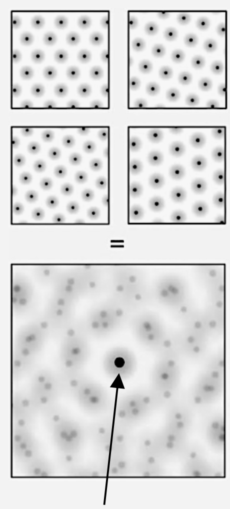
Holographic patterns may even play a role in perception and memory. This idea is explored in the holonomic brain theory posed by neuroscientist Karl Pribram and physicist David Bohm.[218] They proposed that signals in the brain are encoded through holographic interference with network-based properties. This would mean that data is not stored in individual neurons, but in the connections between neurons. Long-term memory is one such example of a network-based storage phenomenon.[219] Experiments have shown that removing small sections of the brain does not destroy certain memories, suggesting that memory in part exists across the neural network. This is similar to a holographic pattern, where the underlying pattern would not be destroyed, but only decreases in resolution, by removing a given piece. Another reason potentially driving holonomic brain processes is that a holographic encoding is extremely energy-efficient and resilient to errors. It is important to note that some brain functions are highly specific to regions of the brain. However, interconnected networks and holographic encoding may still play a critical role in certain functions like memory and perception.

Example 10.10
Hexagon Grid Cells

A neural grid cell fires as an animal crossess roughly hexagonal points on a landscape.



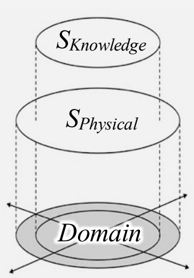
A grid cell represents any junction. To find a unique position, multiple grid cells with different orientations are needed. The four hexagonal grids below, for example, only overlap at one point.



Cognition and Consciousness

Example 10.11
Emergence of Knowledge

In modeling one reality, nature (e.g. humans) has the emergent ability of knowledge, which grows by acquiring information with explanatory power of nature itself. Knowledge allows nature to model itself via sciences. The informational markers of any type of knowledge (or formal system) emerge from physical laws.



While there have been many discoveries in fields such as neuroscience, cognitive science, psychology, and others, the exact nature of what distinguishes a system to be "cognitive" or "conscious" does not have a mutually agreed definition by the scientific community. It is clear that information processes are integrally related to many cognitive activities such as perception, memory, reasoning, knowledge, and a state of awareness. However, exactly what it takes to make an informational or biological system conscious is not well-defined. This can lead to numerous questions, such as whether artificial computers can be considered conscious.

A commonly explored question is what consciousness is made of. Some theories have proposed that consciousness is another type of substance not made of matter, such as the Cartesian mind vs. matter division. A central view of systems science is that matter and mind are interrelated and do not require a harsh division. Emergent properties of matter, like computation or cognition, can have vastly different behaviors than typical matter, but do not require a new type of substance and are considered parallel descriptions of the same reality. So, while conscious systems may have unique properties, a new fundamental substance should not be required to explain them.

A longstanding goal of science is to clarify the criteria for physical systems to have the emergent properties of cognition and consciousness. One may attempt to say that cognitive systems should only arise in animals with sensory organs, nervous system, and brains. However, it is difficult to draw the exact boundary where the brain, and other sensory organs starts and ends. Also, this begs the question if animals without brains can perform cognitive activities, which seems plausible. A broader theory is to associate cognition with any living process down to cellular processes. This follows Maturana's and Varela's Santiago theory that cognition is the process of autopoiesis and is present in all living systems. Following this definition, only information systems present in living systems, such as DNA and neural networks, would be considered cognitive, and nonliving systems like computers would not have cognition. This means cognition is composed of the processes associated with self-making, self-repairing, self-replication, and life. The cognitive information in an autopoietic system would require a self-reflexive and meta quality as it is the information to construct oneself, rather than just the information needed to function a given procedure.

Other theories of consciousness are not restricted to living systems, such as integrated information theory.[220] Integrated information relates to the number of causal relationships between components of a system and has been posed as measure of consciousness. For example, a digital photograph has zero integrated information, because changing the value of one pixel does not influence others. However, changing a node in an artificial neural network can affect how the entire system processes results and causally relates, which means the system has integrated information. Using integration information as a measure for consciousness aligns with studies that the number of neural connections (leading to more integrated information) is correlated to greater cognitive abilities, such as self-awareness. In its most general form, integrated information extends to all forms of matter, like artificial neural networks or the Internet, and is not just limited to biological systems. However, biological systems, such as the brain, would have much more integrated information than typical artificial systems.

Integrated information calculation

- *Partition a system into different components*
- *For each possible state, analyze the causal relations of components*
- *Identify integrated relations that can not be explained by one component alone*
- *Compute the total integrated information of the system*

Figure 10-17
Integrated Information

Summary

Information provides a critical tool for understanding complex systems like genes, brains, and computers. These information interactions can also be entangled, highly interconnected, and produce emergent properties. Computation irreducibility reinforces the ability for complexity to emerge with no ability to compress the data required to model the system. Information also plays a role in perception and cognition, which is particularly interesting, as these phenomena describe the process by which the universe creates models of itself and learns over time.

PART III - APPLICATIONS

Chapter 11 Sustainability

Example 11.1
Renewable Energy

Renewable energy, like wind and solar, creates electricity from resources that continually replenish and supports social, economic, and environmental sustainability.

A highly practical application of system science is sustainability. Sustainable systems maintain and sustain resource reservoirs over many generations. Strategies to improve sustainability include increasing efficiency, using renewable energy, sourcing recyclable resources, and increasing network resilience. Beyond physical resources, sustainability applies to maintaining economic resources, like monetary capital, as well as societal resources, like healthcare and social wellbeing. Altogether, sustainability works to establish and enable the ability for social, environmental, and economic systems to sustain and thrive.

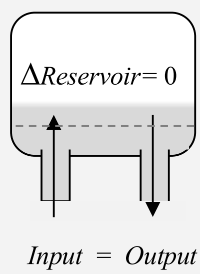
$$S:\{\Delta Reservoirs = 0\}$$

Figure 11-1 Equation for Sustainability

Sustainability has opened a new chapter in the history of science in the 21st century that includes the study of climate change, resource limitations, and global environmental impacts. These findings show that humanity's technologies are interlinked with the planetary environment. Sustainability sciences demonstrate that extractive processes can have negative global impacts the need to design healthy relations between society and the environment to ensure long-term prosperity. To create solutions to complex problems, sustainability draws insights from natural sciences and social sciences as well as employs principles of systems thinking.

Example 11.2
Sustainable Reserves

A sustainable resource reservoir stays constant over time. This means that the input rate is equal to the output rate.



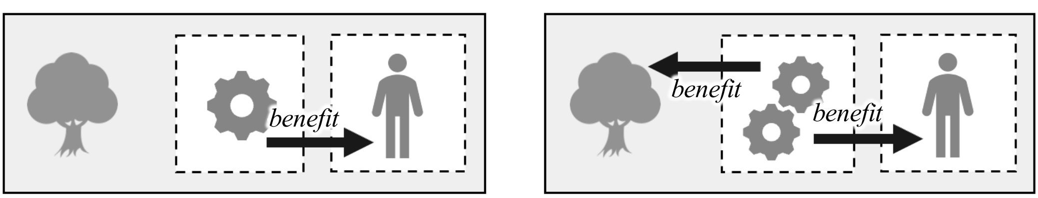
Sustainable Systems

Sustainable systems are designed to persist over many generations. For example, the use of wood resources can be sustained for many generations if wood is cut at a rate equal to or below the forest replenishment rate. Sustainable systems differ from extractive systems, which use resources faster than the rate of replenishment, thereby causing reserves to decline. For example, using wood resources faster than the replenishment rate is extractive and depletes reserves for future generations. Regenerative systems use resources slower than the replenishment rate and allow reserves to grow over time, shown in Figure 11-2. Beyond physical and environmental measures, sustaining and regenerating resources can also apply to social and economic measures.

Regenerative:	*Rate of use < Replenishment*	*Reserves decline*
Sustainable:	*Rate of use = Replenishment*	*Reserves stable*
Extractive:	*Rate of use > Replenishment*	*Reserve grow*

Figure 11-2 Sustainable Reserves

Sustainable designs strive to provide benefits simultaneously for the user and broader world. This differs from an output design that works to optimize the greatest benefit to the user regardless of the environmental and societal impact. Output design can have the unintentional effect of harming the environment and society, as it is only optimized for the beneficiary. In contrast, sustainable designs work to simultaneously optimize benefits for the agent as well as the broader world. Benefiting the larger system ensures the ability to sustain the external supporting factors that enables the technology to function. Sustainable design acknowledges that users are not separate from the environment and finds optimal, systems-wide, solutions.



Output Design: *(Beneficiary optimization)*   Sustainable Design: *(System-wide optimization)*

Figure 11-3 Output vs. Sustainable Design

Nature-Inspired Designs

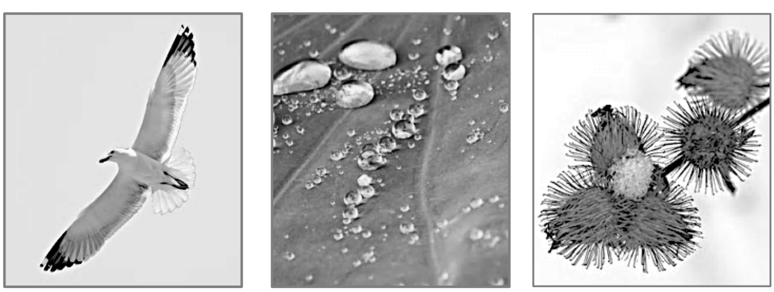
One strategy to develop sustainable designs is to find inspiration in nature itself. A prime example is that physical systems tend to follow the principle of least action and minimize energy use, like water flowing down a hill on the path of least resistance or atoms creating efficient crystal geometries. Using structural designs that follow the path of least resistance, such as catenary curves, minimal surfaces, and geodesic domes, can support sustainability by using less resources to achieve useful outcomes.

Living systems provide many useful insights for sustainable design. The field of biomimicry utilizes insights from biology to address complex engineering problems, such as locomotion, structure, and growth. Biomimicry is particularly relevant for sustainability because living systems have evolved over millions of years to utilize designs that are fully recyclable, resource efficient, resilient, and adaptive to change. Life as a whole is required to be sustainable in some form, otherwise life would fail to sustain over new generations and cease to exist.

Many technologies draw inspiration from biology. For example, airplane wings generate lift in a way similar to bird wings. Water repellent coatings make use of small protruding bumps to condense water into droplets in a fashion similar to the water lily. Velcro mimics the burr fruit's use of hooks to adhere to surfaces, as displayed on Figure 11-4. Even unintuitive patterns in nature can prove to be beneficial. For example, the small protruding bumps on whale fins would seem to increase drag, but actually increase efficiency for long distance swims. Similar bumps can be used to improve wind turbine performance.[221]

Example 11.3
Learning from Termites

Termite nests provides natural ventilation by the surrounding air pressure and orientation to the sun. A similar technique was used in the East Gate building in Zimbabwe for energy efficient ventilation.



Plane flight and bird | *Water repellent materials and lilies* | *Velcro and plant hooks*

Figure 11-4 Biomimicry Design Examples

Ecosystems are able to sustain life for many generations without running out of energy or materials, providing insights for sustainable design principles. For example, many plants use renewable energy resources, like the Sun, to sustain over generations versus short-lived energy resources. Likewise, the use of renewable energy is a core principle of sustainable design. Ecosystems also exist in a fully recyclable biosphere, or else materials would run out over time. This is analogous to the sustainability goal of using recyclable materials. Additionally, ecosystems tend to organize into resilient networks, to deal with volatility in the environment, as well as evolve over time to better suit the environment. Resilient networks and adaptive evolution are also useful designs to cope with changes in in resource exchange networks (e.g. power, water, transportation, economic systems). A summary of these ecologically-inspired sustainability design principles is given in Figure 11-5.

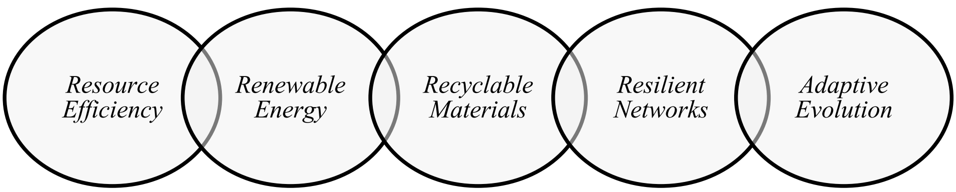


Figure 11-5 Sustainability & Ecosystems Design Principles

Figure 11-6 Biophilic Design

Biophilic design is another nature-inspired tactic that incorporates natural elements in architecture. The biophilia hypothesis, posed by Edward Wilson in 1984, posits that people possess an innate tendency to seek connection with nature and life. Biophilic design supports the human connection to life by incorporating plants, natural light, airflow, water, Earth-friendly materials, and biomorphic forms into architecture. Studies show that there are positive cognitive, health, and economic benefits for people residing in buildings utilizing natural elements.[222] For example, hospital patients with plants in their rooms have shown better recovery rates.[223] Figure 11-6 shows a biophilic design where a sky-scraper has trees on all the balconies, integrating nature with living spaces.

Resource Efficiency

Efficiency supports sustainable design by minimizing the resources and energy required to achieve desired outcomes. Efficiency is defined as the ratio of useful work to total energy used. Efficiency is increased by gaining more useful work or decreasing the wasted energy. An example of energy efficiency would be to use less energy to achieve the same amount of heating outputs to the user, as displayed in Figure 11-7. This could be achieved by better insulation, closing air gaps, or installing windows that retain more heat. Sustainable design strategies leverage energy efficiency as well as material efficiency in producing goods and services.

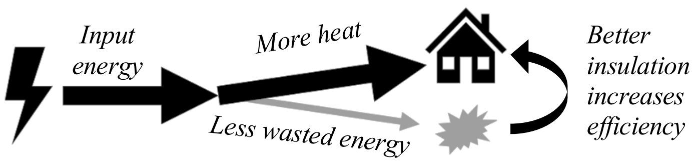


Figure 11-7 Improving Efficiency

Currently, the world is in an energy efficiency revolution. For the first time in modern history, developed economies are reducing total energy use. This is largely due to efficient lighting, heating, and transportation systems. The decrease in energy use has also occurred during an increase in economic growth. From 2007 to 2022, the total energy use of the OCED countries decreased by 7.7%, while the total value of produced goods and services (GDP) increased by 82%.[224] More energy is not always required to increase financial outcomes and it is possible to do more with less. As expounded upon by U.S. President Obama, prioritizing energy efficiency can support long-term economic value generation.[225] Applying energy efficiency at global scales is critical to reduce extraction, sustain reservoirs, and gain value.

An important efficiency consideration is that avoiding the need to use energy in the first place is the most sustainable energy strategy. For example, driving the most energy-efficient car still uses more energy than not driving any car. Having efficient devices should not be used as an excuse to use technologies that are not required in the first place. Additionally, the kinds of energy inputs that are used are critically important. Sustainable designs prioritize pairing efficiency with renewable resources.

Example 11.4
Efficient Paths

The traveling salesperson problem asks what is the shortest possible path to visit each city once and return to the origin, such as in the graph below.

The problem is easy to solve for a low number of cities, but there is no quick way to solve for many cities. Efficiently managing resources, supply chains, and other economic systems can lead to complex problems.

Renewable Energy

Example 11.5
Renewable Goals

Over 50 countries have committed to 100% renewable energy by 2050, including Denmark, Sweden, Costa Rica, Morocco, and Sri Lanka. Some states in the U.S. also have this goal, like Maine, California, Hawaii, and Nevada.

The use of renewable energy is a critical dimension of sustainable systems. Society uses a broad array of technologies for renewable energy sources, like solar, wind, and hydroelectric, as well as non-renewable sources, like fossil fuels and nuclear energy. The continuous power provided by renewable energy sources can be measured in terawatts TW (one trillion watts). Non-renewable energy, like coal, oil, gas, and uranium have finite reservoirs, very low replenishment rates, and are measured in terawatt-years TWy, the energy that can be continuously provided for one year.

Global estimated of renewable and non-renewable energy resources are summarized in Figure 11-8. Renewable energy resources are much larger than fossil fuel reserves and can power the world many times over. For example, solar energy provides 23,000 TW continuously year over year, while the entire world's fossil fuels and uranium can only provide 1,570 TW for one year. With the 2021 global energy use at ~20 TW, finite energy sources could only fully power the world for ~80 years at current rates.

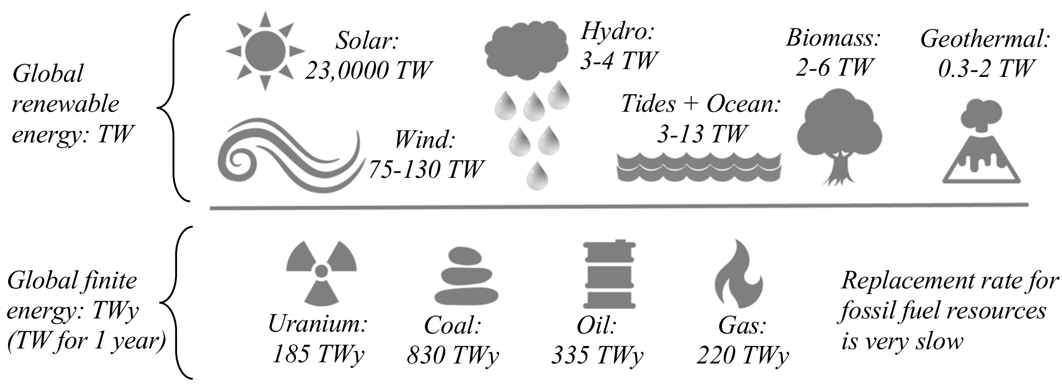


Figure 11-8 Global Renewable and Finite Energy Sources

The 21st century marks a pivotal transition where renewable energy sources, like wind and solar, are more cost effective than fossil fuels—the dominant source of energy since the industrial revolution, which is finite and severely impacting the climate. Utility-scale solar and wind power plants have been the lowest cost of new energy as far back as 2015, and renewable prices continue to fall.[226] Improvements in batteries are also supporting the ability to power electric vehicles and the electric grid. Going into the 21st century, it is now feasible to power the world with 100% renewable energy. Enacting this change is necessary to power humanity for many generations.

Regenerative Food

Regenerative agricultural methods are a critical dimension of sustainability. Conventional use of monocrops, synthetic fertilizers, and pesticides have been shown to be able to produce increased short-term results, but they can lead to long-term negative effects, like soil erosion. Huge desertification events like the 1930s Dust Bowl in the U.S. were exacerbated by extractive agriculture methods. [227] In contrast, regenerative agriculture techniques, like no-till farming, planting polycultures, and natural fertilizers can improve soil and reduce long-term value.[228] Regenerative land management supports increasing the capture of carbon dioxide through plants and compost, which reduces climate change impacts. Regenerative agriculture works for long-term and system-wide benefits for water, carbon, soil, and ecological resilience, with benefits summarized in Figure 11-9.

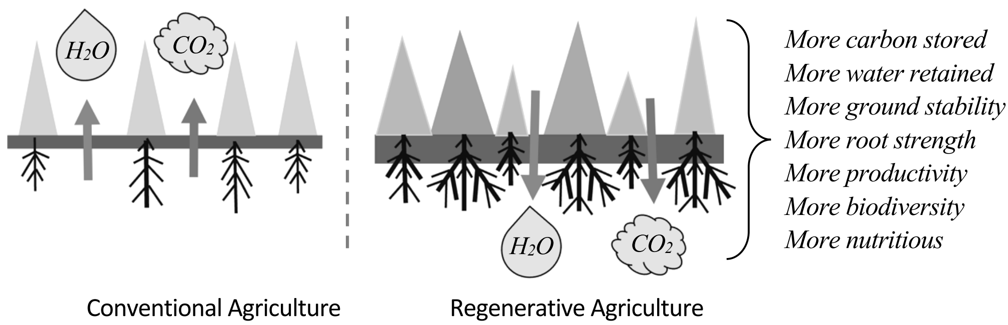


Figure 11-9 Sustainable Agricultural Management

There are many ways to increase the responsibility and sustainability of food systems. Across the world, and especially in industrialized countries, the consumption of high amounts of meat has immense environmental impacts. Reducing or eliminating consumption of unsustainable meat is one of the most impactful ways to reduce environmental impacts. Specific food products can be linked with outsized negative environmental impacts, such as palm oil linkage to deforestation. Shopping locally and regionally is an important piece of supporting sustainable food systems and presents a number of benefits such as lower transportation distances.

Food systems are deeply related to personal health and public wellness. For example, eating processed sugar is correlated with negative health outcomes, like obesity and diabetes.[229] Food can also support healthy outcomes, such as turmeric with black pepper, which can support reducing inflammation symptoms.[230] Designing healthy food systems is critical to maintaining social wellbeing.

Sustainable Structures

> **Example 11.6**
> Earthships
>
> Earthships are highly sustainble buildings that follow the six principles of:
>
> *1) passive solar heating and cooling*
>
> *2) solar & wind electricity*
>
> *3) natural and recycled materials*
>
> *4) water harvesting*
>
> *5) food production*
>
> *6) water treatment*

Buildings use a large quantity of resources and utilizing designs that can efficiently provide heating, electricity, water, and other essential functions is an avenue to support sustainability. Sustainable buildings not only save energy and benefit the environment but can also save money and support positive social outcomes. Sustainable design can be applied to all types of buildings, from residential houses to commercial factories to entire cities.

Sustainable structures utilize many techniques to minimize inputs, like using solar energy, collecting water, and growing food. This reduces the need to provide external resource inputs. For example, buildings can save energy by being oriented in a fashion to allow passive solar light to heat the building floors during winter and stay in the shade in summer. Rooftop solar panels can be used to generate renewable energy and reduce electricity costs. Rainwater collection and secondary reuse of water are other important strategies to enable water efficiency for the household and other uses, like gardening. Onsite composting is another tool that can be used to recycle food waste and generate soil for local gardens. Some of these features of sustainable buildings are summarized in Figure 11-10. As a total system, sustainable structures provide shelter, heat, electricity, water, and food with minimal to zero resources beyond the structure and surrounding environment.

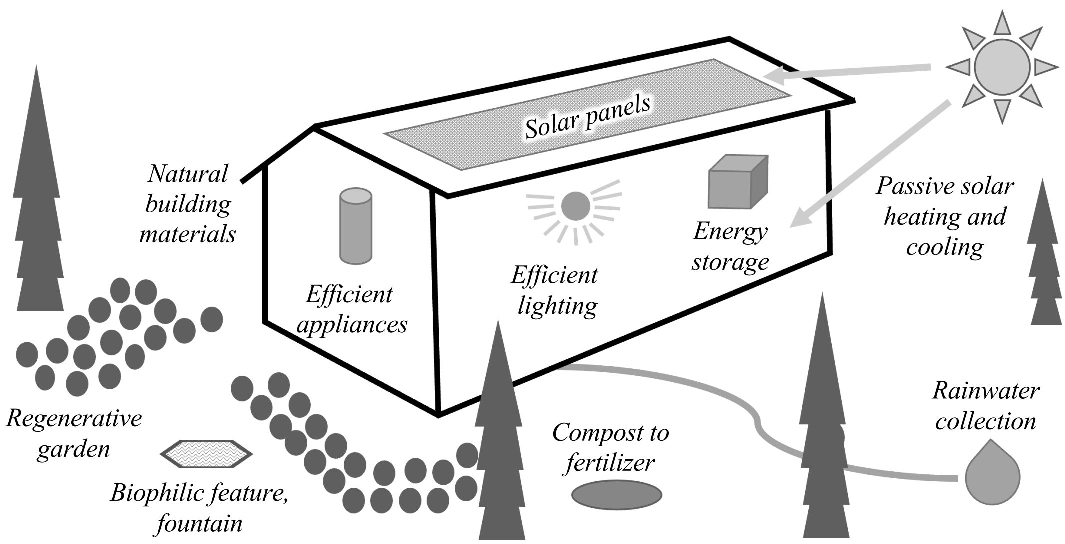


Figure 11-10 Sustainable Building Features

Recyclable Materials

Recycling transforms used products into new products, reducing the need to extract resources. Currently, most manufactured products have low rates of recycling and end their lifecyle in the landfill, which creates a linear flow from extraction to waste. In contrast, a circular economy prioritizes high rates of recycling, thereby lowering the need to extract new resources or discard resources for no further use. Many methods can be used to increase circularity, like reducing, reusing, and repairing products.[231] In a fully circular economy, all resources are recycled and reused.

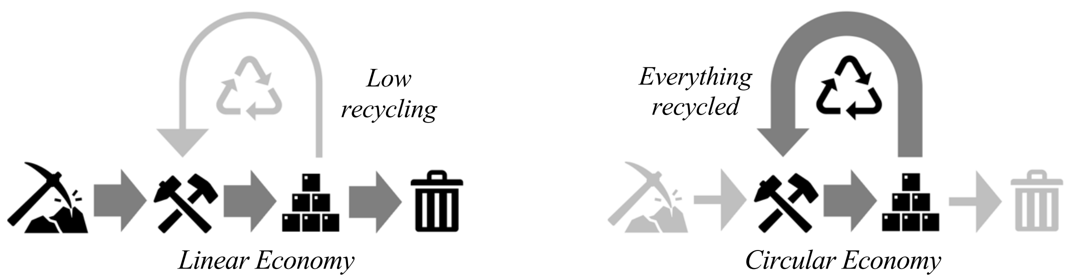


Figure 11-11 Linear vs. Circular Economy

A fully zero waste economy takes strategic planning. From the start, products need to be designed to be completely recyclable at the end of use. Creating biodegradable goods is one such method. Establishing municipal waste systems with enough capacity to process materials is another necessary component to achieve zero waste. Many stakeholders, from industry to government to nonprofits, need to work together to create a fully circular economy. While achieving net zero waste may sound like a lofty goal, it is a requirement for long-term sustainability (landfills cannot be filled forever) and natural ecosystems are already fully recyclable and reuse all resources. If ecosystems can be zero waste, so can humanity.

Recent technological advances are creating entirely novel methods to recycle and manufacture materials. For example, the ProtoCycler is a 3-D printer that can grind down existing plastic items to be melted and printed into new forms.[232] In the future, it might be common to print products through fully recycled materials that are sourced locally, minimizing resource use and transportation. Other new technologies, like plant-based polymers and reusable packages, are making a circular economy a reality for many goods.

Resilient Networks

Example 11.7
Network Design

In centralized networks all nodes are connected to a hub, increasing efficiency. A distributed network has no hubs, increasing resiliency. Decentralized networks have multiple hubs, that strike a middle ground of efficiency and resilience. Utilizing the appropriate network design is critical for sustainable design.

Centralized

Decentralized

Distributed

Sustainable systems often require optimizing how components in a network synergistically interact. For example, urban planning needs to optimize multiple interconnected networks, like transportation, energy, and water. Sustainable urban design works to increase efficiencies and promote the resilience to unexpected changes. Many types of planners need to work together to optimize networks, like transportation routes, energy grids, and water distribution, to support full-scale sustainability.

Electric grids can benefit from resilient network design. Traditional electric systems were designed as centralized top-down networks where large power plants sent electricity to consumers. Modern smart grids now enable two-way flows of energy in a decentralized network. Smart grids optimize power plants, residential solar, batteries, electric cars, and other energy resources in a collective network, as shown in Figure 11-12. Decentralized grids can promote local renewable energy integration and are more resilient to unexpected disturbances.[233]

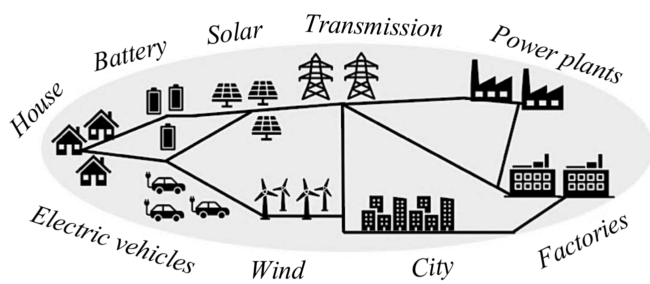


Figure 11-12 Smart Grid Networks

Optimizing networks for sustainable outcomes often involves balancing efficiency and resilience. For example, one large highway might be the most efficient route between two cities when operating at full capacity, but a network of smaller roads can be more resilient to unexpected closures. Biomimicry can offer insights to find ways to optimize networks across multiple factors. Models of ant transportation routes, which are both efficient and robust against disruptions, can help design better human-scale transportation networks between hubs of activity.[234] Slime mold has even been shown to grow networks similar to the efficient Tokyo subway system when pieces of food are places to mimic the position of city destinations.[235]

Economic Sustainability

Supporting just and resilient economic systems is a pillar of sustainability. The economy is a system of interrelated production and consumption transactions influencing how goods, services, and resources are allocated. Economics studies how measures like supply, demand, and price relate to one another. Sustainable economic systems work to conserve and improve financial, social, and environmental resources over the long-term.

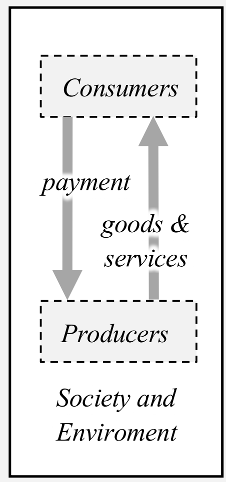
One dimension of economic systems is the degree of choice individuals have in making decisions. In a purely free market, there are no collective rules dictating individual transactions, while planned economies establish centralized rules that apply to many people. One pitfall of no regulations is the "tragedy of the commons", where individuals extract resources for personal value, but at the expense of diminishing the collectively shared value.[236] On the other hand, too much government regulation can reduce the ability for local groups to innovate solutions. Most economies in the world today exist as some combination of the two. Network models of economic systems can provide a toolset for policy makers to make the best decisions that optimize for both individual and group decisions.[237]

Some ideas associated with capitalism, like infinite growth by depleting environmental resources, need to be transformed to align with sustainability. It is impossible for extractive industries to grow forever on a planet with finite resources. Long-term sustainable economic value must be driven by the efficient use of resources, renewable energy, and recyclable materials. Low-cost energy efficiency and renewable technologies are already increasing value while reducing resource extraction.

Another capitalistic concept not aligned with sustainability is that a company's sole mission is to create profit for shareholders. While often believed to be true, U.S. corporate law does not dictate that companies must maximize profits or share price.[238] To the contrary, company leaders are permitted to make decisions that are in the best interest of many other stakeholders, such as employees, customers, and society. Companies often reduce short-term profit gains for shareholders with no legal repercussions, such as increasing pay to employees, conducting more research and development, or donating to non-profits. A company's purpose is to provide long-term value to many stakeholders, not just generate quick profits for shareholders. Debunking the shareholder value myth is an essential ideological change to support a sustainable, systems view, of economics that integrates social and ecological factors in decisions.

Example 11.8
Economic Systems

The economy is a system of consumers and producers that transact with financial payments and goods. The economy works within society and the environment.



Another aspect of sustainable economics is expanding the scope of capital. Capital is often thought of as just financial assets, but other forms of capital exist. Social capital and natural capital may not have direct financial equivalences but are nonetheless valuable. Social capital, for example, can include intellectual expertise that may not be on a balance sheet, but is essential for generating value. An example of natural capital is services provided to society by ecosystems. For example, wetlands can provide economic value by cleaning water, which would cost money to accomplish in a water treatment plant if the ecosystem were destroyed. Sustainable economics works to optimize all types of financial, social, and natural capital, some of which are listed in Figure 11-13.

Financial Capital

- *Cash*
- *Property*
- *Investments*
- *Debt and loans*

Social Capital

- *Employee skills*
- *Social organizing*
- *Intellectual skills*
- *Community support*

Natural Capital

- *Ecosystem services*
- *Eco-tourism*
- *Ecological products*
- *Resource production*

Figure 11-13 Multiple Types of Capital

Sustainable economics considers forms of social capital that may be entirely omitted in traditional economics. Approximately 60% of the global working population is part of the informal economy that exchanges goods and services not monitored by governments.[239] For example, many people in developing countries provide food, water, and energy for their households with no formal capital exchange. While capitalistic endeavors may raise monetary accounts on paper, these actions may actually diminish the standard of living. Sustainable economics takes a system-wide approach to make decisions that provide benefits across financial, social, and natural forms of capital.

Traditional economic metrics like Gross Domestic Product (GDP) should be used with caution when assessing sustainable outcomes. GDP measures the total amount of money spent on goods and services in a country over a specific time frame, with no consideration of whether the goods and services were used for positive or negative outcomes. For example, spending money on unnecessary military equipment or to clean up an oil spill will raise GDP. Also, GDP can grow by increasing profits for the wealthy, while the majority of a population makes less money. Many organizations are aligned that GDP should not be considered an effective indicator to measure a country's standard of living or assess economic well-being.[240] Other metrics such as wealth distribution, income after living expenses, and public health indicators are more relevant to assess sustainability progress and societal wellbeing. Replacing, or augmenting, GDP to include other social and environmental factors is important to assess sustainable economic outcomes.

Another misleading metric for sustainable economics is net present value. Net present value compares future earnings to present earnings using a growth or discount rate. For example, if money is expected to grow at a 10% rate, then $100 today should be worth $110 dollars in one year. Following this 10% growth rate, receiving $100 now would be better that receiving $105 in one year, because the $100 is expected to grow to $110. However, receiving $120 one year from now would net someone more value than $100 today because, following the same 10% discount rate, $120 in one year would equal $109 of net present value, calculated as $120 / (100% + 10%) = $109.

A net present value analysis can be helpful for simple, short-term, tradeoffs, but there can be unsustainable repercussions from this type of analysis. For example, consider a forest that could either produce $100 of sustainable wood products per year, or could be cut down and sold for $2,000. Following a discount rate of 10%, the net present value of all future years is $1100 ($100 + $92 + $84 +…) and lower than $2000. While cutting down all the trees may seem like the better option using net present value, the model neglects the fact that after 20 years the forest will provide more that $2,000 and continue to do so every year after as a sustainable asset. Discounting the future through net present value can lead to the prioritization of short-term gains over long-term sustainable outcomes.

To more accurately measure sustainable economic outcomes, a whole host of new company disclosures have been introduced in the 21st century. Many of the largest U.S. companies now disclose sustainability reports detailing social, environmental, and economic outcomes. These disclosures can include carbon emissions, environmental mitigation, worker safety, community engagement, and other metrics outlined in Figure 11-14. Some companies even publish integrated annual reports that include financial and sustainability metrics side-by-side. Regulators and advocates are working to improve the standardization and comparability of these system-wide measures.

The 21st century marks a revolutionary shift, where investors are now taking sustainable economics seriously. As of 2022, 13% of the professionally managed assets in the U.S. include sustainable measures in their investment approach.[241] New investment funds have been created that are dedicated to optimizing environmental, social, and governance (ESG) outcomes. Many of these funds even correlate with increased financial returns. For example, from December 2013 to December 2023, the 500 largest U.S. companies grew 165% while the 500 largest U.S. companies that exclude fossil fuel companies grew 178%.[242] This result, among others, shows that ESG optimization can support long-term financial return. This evidence counters traditional portfolio theory that says more diversification is always better. It may be the case that sustainability leaders outperform the market over the long-term.

Environmental

- Carbon emissions
- Resource efficiency
- Efficient water usage
- Supplier standards

Social

- Worker safety
- Health coverage
- Fair trade
- User satisfaction

Governance

- Meeting regulations
- Lobbying expenses
- CEO-to-worker pay
- Worker pay metrics

Figure 11-14
Sustainability Metrics

Social Sustainability

Social wellbeing is a pillar of sustainability alongside the economy and environment. Social sustainability enables current and future generations to create healthy and livable communities. Supporting equitable, diverse, and connected societies is essential for social sustainability, well-being, and resilience. Additionally, social sustainability entails improving a host of public health outcomes, such as reducing infant mortality and exposure to curable illnesses.

While an important topic in its own right, social wellbeing is deeply linked to the management of ecological and economic resources. Access to food, water, resources, and energy, allows society to thrive. Economic measures, like wealth distribution, minimum wage, and cost of living, are integrally related to social wellbeing. Social wellbeing is interlinked with ecological and economic resources and must be managed as a connected system.

Diversity plays an important role in social resilience, adaptation, and innovation. The benefits of diversity can be illustrated in an example from agriculture. Farms with monocrops and fewer types of plants have a much harder time surviving disruptive changes compared to a diverse collection of crops. Similarly, creating monocultures of ethnicity, religion, and culture hinders a society's ability to innovate and consider different views. Studies also show that more culturally diverse social communities are able to better connect and form bridges with other communities.[243] Building social bridges is essential to create resilient networks that are able to pool resources and share ideas beyond the immediate group.

Cherishing diversity is essential to supporting social sustainability and creating strong social networks. This is extremely important as there has been a history of social, political, and economic oppression to exploit diverse cultures with negative outcomes for social wellbeing and quality of life. Social sustainability works to find ways for diverse groups of people to collectively work together and support one another to become more resilient. A systems perspective acknowledges that all humans are on Earth together and that our actions are interrelated. As global citizens, mental frameworks need to transition to appreciate diversity and unique perspectives.

Example 11.9
Small World Networks

Social networks are often small world networks, where distant strangers are linked by a short chain. For example, most people on Earth only have six degrees of separation. Small world networks cluster in cliques, or groups, with some key connection bridges. The average length between nodes in a small world network follows a scale-free logarithmic ratio.

Length $\propto$ *Log(Nodes)*

Another way to promote resilience in an organization is to increase autonomy of subgroups by transitioning away from a purely top-down, need-to-know hierarchy. For example, instead of one leader that makes all decisions, it is possible to create local groups that collaborate to reach decisions within a larger collective. In this model, the role of leaders is less about dictating rules and instead facilitating communication, coordination, and innovation. The snowflake organization is one such model relevant for businesses and advocacy groups of how many smaller groups can work together to reach a collective goal. U.S. President Barack Obama's presidential campaign utilized this snowflake model; instead of all ideas being presented in a top-down fashion, ideas were generated at all levels of the organization.[244] This model can help combine the benefits of both decentralized creativity and centralized efficiency.

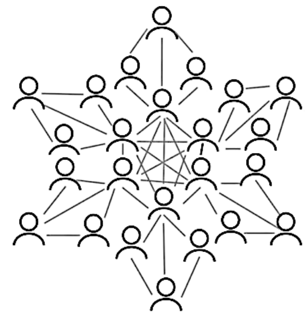


Figure 11-15
Snowflake Model

Political systems can greatly influence society's ability to sustain over time or change to meet new conditions. Civic rights such as education, free speech, gender equality, and fair legal processes play an important role in supporting healthy and resilient communities. Government regulations span beyond social issues and can influence the management of ecological resources. Governments can establish environmental policies, like carbon emission reduction goals, wildlife protections, renewable energy targets, and clean water standards, to support sustainable outcomes. Sustainable political institutions promote social and economic justice as well as the responsible management and use of ecological resources.

Another critical component of social sustainability is reducing and ending violence and warfare. Firstly, military conflict has immense negative societal impacts and has caused harm to millions of humans throughout history. Furthermore, warfare is extremely resource-intensive in terms of ecological and economic measures and does not contribute to sustainable systems. Wars are often used as tools to enforce unsustainable behaviors, like negative social hierarchies, taking of resources at the expense of others, or limiting of freedom of thought. Living organisms are not meant to destroy life on a global and country-wide scale. In ecosystems, different groups of animals may compete with one another, but no animal commit acts of war at the scale of humans. Life's evolutionary goal is to support life, not destroy it. Human competition should take the form of innovating ideas for wellbeing and not in warfare and violence to control others.

Global-to-Personal Awareness

Countries need to work together and set comprehensive priorities and actions to achieve sustainability on a global scale. One such example of progress toward global sustainability awareness and coordinated action is the United Nations' Sustainable Development Goals. These goals set ambitious targets for countries, companies, and non-profits to sustain resources and promote social wellbeing. The sustainable development goals include no poverty, zero hunger, gender equality, clean water, quality education, climate action, and other actions listed on Figure 11-16.[245] These goals span environmental, economic, and social issues and emphasize the necessity to take a cross-sectional, systems-based, approach to create sustainable solutions. Beyond these goals, other targets and accountability frameworks should be created to shift exploitative processes to regenerative alternatives at a global scale.

U.N. Sustainable Development Goals	*1: No Poverty*	*2: Zero Hunger*	*3: Good Health and Wellbeing*	*4: Quality Education*	*5: Gender Equality*
6: Clean Water and Sanitation	*7: Affordable and Clean Energy*	*8: Decent Work and Economic Growth*	*9: Industry, Innovation and Infrastructure*	*10: Reduced Inequalities*	*11: Sustainable Cities and Communities*
12: Responsible Consumption and Production	*13: Climate Action*	*14: Life Below Water*	*15: Life on Land*	*16: Peace, Justice and Strong Institutions*	*17: Partnership for the Goals*

Figure 11-16 U.N. Sustainable Development Goals

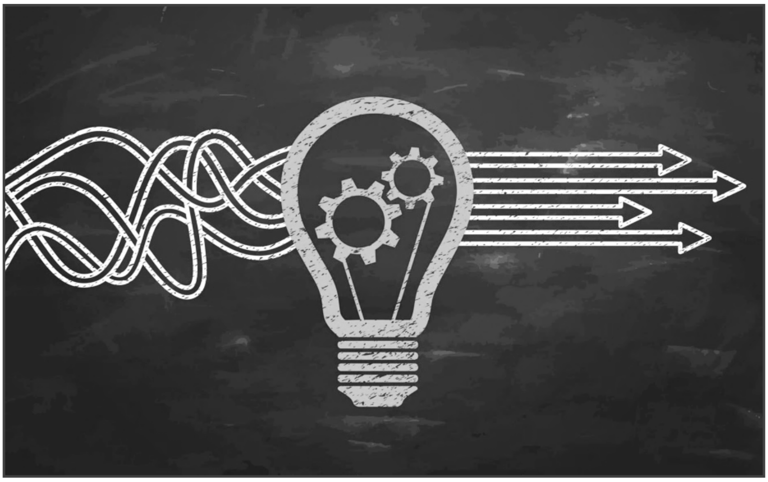
Sustainability presents a new paradigm that departs from extractive industrialism and focuses on designing for longevity. Some thinkers have marked the sustainability movement as a revolutionary turning point in humanity that encompasses a new worldview and era of technology. For example, Joanna Macy uses the term “The Great Turning” to represent the global transition following the Industrial Revolution, marked by eco-friendly technology and global sustainability awareness.[246] This Great Turning is characterized by life-sustaining systems and practices, like the widespread use of renewable energy and recyclable materials, as well as a shift in thinking, values, and a raised awareness of social and environmental responsibility.

The process of working to fix large-scale problems should always be balanced with ensuring personal sustainability. It is essential to take care of oneself before prioritizing large-scale solutions. All too often, professionals who focus on sustainability exhibit an unsustainable work-life balance which reduces the quality of their work. People are part of nature, so supporting natural sustainability also means supporting the sustainability of individuals and groups of people. It is important to live by example when approaching sustainability by creating rejuvenating and regenerative personal habits when working on global solutions.

Summary

Altogether, sustainability articulates a new dimension of science and systems thinking. Disciplines like climate change, renewable energy, and sustainable design require understanding the world in terms of complex and interconnected systems. Sustainability demonstrates that human social and economic activities are highly intertwined with global environmental resources. Sustainability uses a systems theory approach by working across disciplines and optimizing the relationships between physical, biological, and information networks. Understanding systems science is a critical tool to analyze the convoluted problems facing the world to design efficient and resilient solutions that will serve humanity for many future generations to come.

Chapter 12 Transformation



Example 12.1 Implementing Change

Agents can change systems in nature and implement different physical technologies and abstract models.

Systems science provides insights to understand transformation, the process by which a system is open to change under the transformation $S_{Present} \rightarrow S_{Future}$. Physical systems can be transformed by changing the underlying elements and relational rules of interaction via energetic and informational processes. Abstract systems can also be transformed through creating new models of the world. For example, scientific frameworks are under continual transformation to account for new evidence. Revolutionary scientific model shifts in the 20th and 21st century surround the inclusion of chaotic and complex systems.

$$S_{Present} \rightarrow S_{Future}$$

Figure 12-1 Equation for Transformation

Understanding how systems can be transformed is critical to enact effective solutions in our modern world. For example, the field of systems engineering studies how many components work together to create synergistic outputs. Another method to improve the process of transformation is a systems-based analysis, which studies the agents and relationships involved in a system to determine the most strategic methods to intervene. Systems analysis can be useful to break apart convoluted political or social problems and create custom-tailored solutions. This approach, when applied to the broader world, can help identify the most strategic ways to enact systemic change.

Transforming Systems

Systems can be transformed by adding, removing, or changing the elements and relations between elements. In the system of a fish tank, for example, changing the quantity of light or food inputs can result in a transformation of the fish population or algae growth. Another way to transform a system would be to add new elements or agents, such as adding a new fish species, which can lead to vastly different results over time. The general process of transformation occurs when a system's elements and relations are exposed to change, as shown in Figure 12-2. In physical systems, transformations are typically accomplished by altering matter, energy, or information.

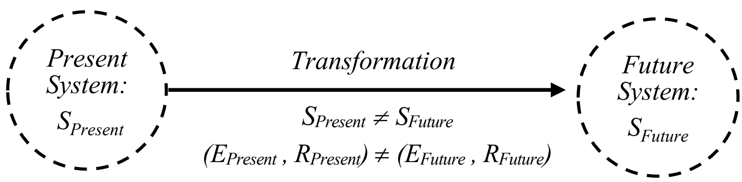


Figure 12-2 Transforming a System

A systems change approach strategically shifts components of complex systems for large impacts. A systems-based view considers nonlinear and interrelated networks instead of focusing on how one input can linearly change one output. For example, consider the problem of a crowded freeway. One solution could be to build another freeway. However, this might add to other problems like congestion and air pollution. It may be better to create public transportation options to walkable city centers to reduce the need for car traffic altogether and make the existing freeway more effective. Many complex problems facing the 21st century world require a systems change perspective that considers interconnections, root problems, leverage points, and other strategies shown in Figure 12-3.

- *Understanding interconnections within and between systems*
- *Seeing patterns of change over time rather than static snapshots*
- *Fixing root problems and avoiding unforeseen consequences*
- *Determining leverage points and change agents for action*
- *Constantly learning, adapting, and probing assumptions*

Figure 12-3 Systems Change Strategies

Systems Engineering

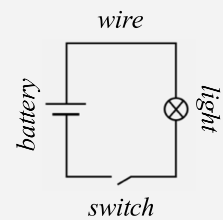
Engineering utilizes scientific principles to design new machines, technologies, and physical systems to perform useful work in the world. These technologies are able to create emergent behaviors, such as locomotion, illumination, computation, and other useful outputs. Many engineering problems, like levers, are fairly simple and only require adding a few components together in a linear fashion. However, engineering problems with many pieces can be highly complex, nonlinear, and require a systems view.

Systems engineering manages how many components work together in a system to accomplish a goal. Systems engineering can include structural, mechanical, electrical, software, and other levels that enable coordinated action. The International Space Station is an example of a complex project that requires systems engineering.[247] This is because there are many kinds of co-existing systems to produce air, water, and light for a habitable environment.

Block diagrams can be used to model engineering systems. A functional block diagram represents a system's functions, or transformational processes, as blocks. The inputs and outputs of the functions are represented by arrow lines. A functional block diagram models how a sequence of interwoven steps work together in a connected system. For example, Figure 12-4 is a functional block diagram for an off-grid sustainable house. This system has input of rainwater and sunlight that is processed into usable resources of heat, electricity, water, and food for the building occupants. These kinds of graphs, and other diagrams, can be translated to rigorous mathematical functions through category theoretic objects and morphisms.

Example 12.2
Circuit Diagrams

Electrical circuit diagrams are graphs used to predict electrical behavior. A diagram of a battery, light, and switch is below.



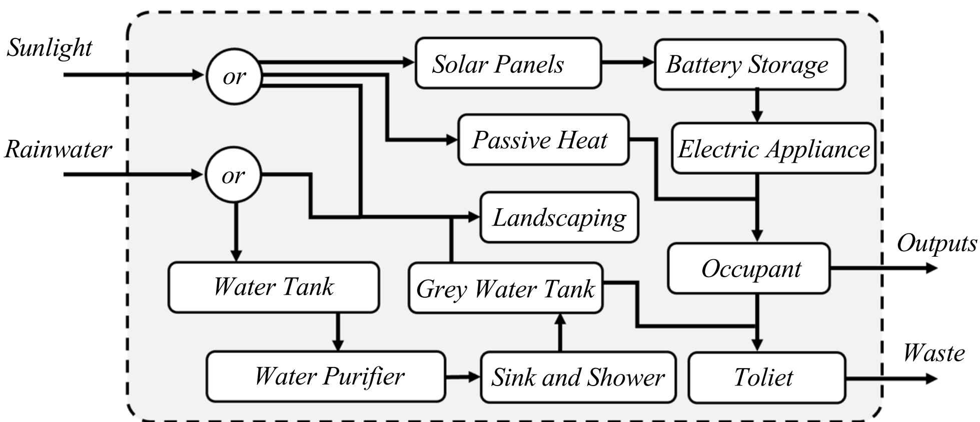


Figure 12-4 Off-Grid House Functional Block Diagram

Data flow diagrams provide a tool to model the information flows and networks in a data system. In data flow diagrams, databases are represented by two lines and the input or output data sources are squares. The data flows are represented by arrows, and the functions, or processes, are circles. In an online business for example, the online shop is the interface that transforms customer inputs into a list of orders from a product list. After an ordered is entered, the database of orders then goes into a fulfillment process, which creates a list of fulfillments carried out by shippers, as shown in Figure 12-5. Data diagrams are useful to build efficient and optimal processes.

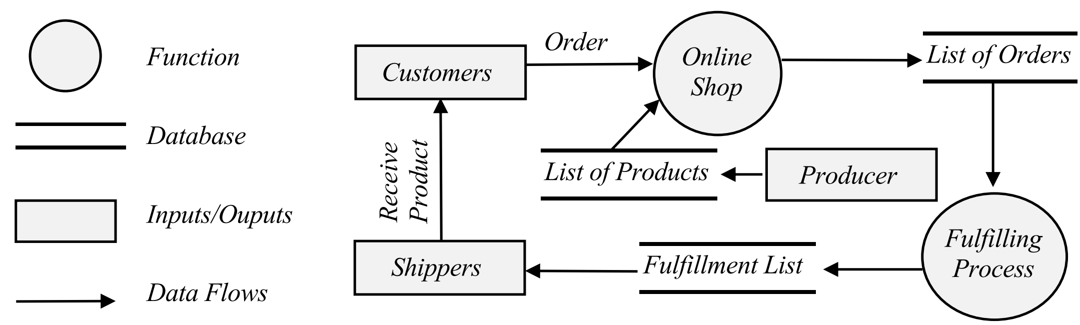


Figure 12-5 Data Flow Diagram

Another way to express engineering functions is an N^2 chart. An N^2 chart is a square matrix where the diagonal cells such as *(1, 1)*, *(2, 2)* or *(3, 3)* represent different functions. The other cells describe how these various functions relate to one another. For example, cell *(1, 2)* represents how Function 1 relates to Function 2 *(F1 → F2)*. These charts can model how all functions relate to one another and provide insights into feedback loops and overall throughput. For example, the functions and feedback processes required for successful aircraft flight, like data sensors, propulsion engineers, and air pressure regulators, can be modeled in an N^2 chart, as each function relates to other functions in unique ways with various forms of feedback.

Example 12.3
Sensitivity Analysis

The robustness of a system can be tested by the total effect of changing one variable at a time. Variables with greater influence to the total have more sensitivity. This analysis can inform resilient designs.

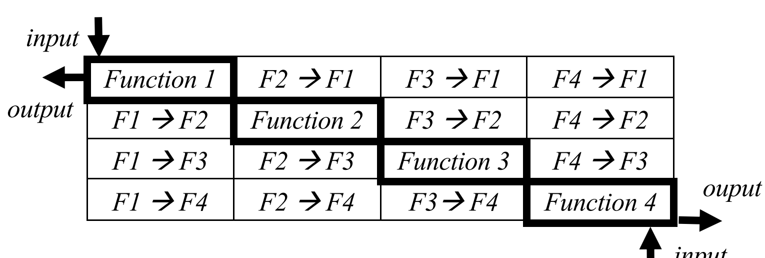


Figure 12-6 N^2 Chart of Multiple Functions

Systems Design

There are various stages of implementing designs from a systems-based perspective that can achieve effective transformations in the world. This process can be summarized in progressive steps, such as: define the task, conceptualize solutions, design prototypes, and implement the final product. An important consideration when defining the problem and conceptualizing solutions is to take the necessary steps to ensure a solution is addressing a need. Some products are designed without regard to addressing human problems or without regard to a full range of human interactions, like the ease of use, relevance, and other practical factors.

Human-centered design uses customer interactions early on to inform the best way to implement solutions. This shift is important to create relevant and effective solutions. This process first begins with interviewing and shadowing users to understand the context and identify relevant problems. After considering user feedback to define challenges and opportunities, different ideas are then prototyped and shared for follow up tests and feedback to create the final product. The human-centered design process is summarized in Figure 12-7.[248]

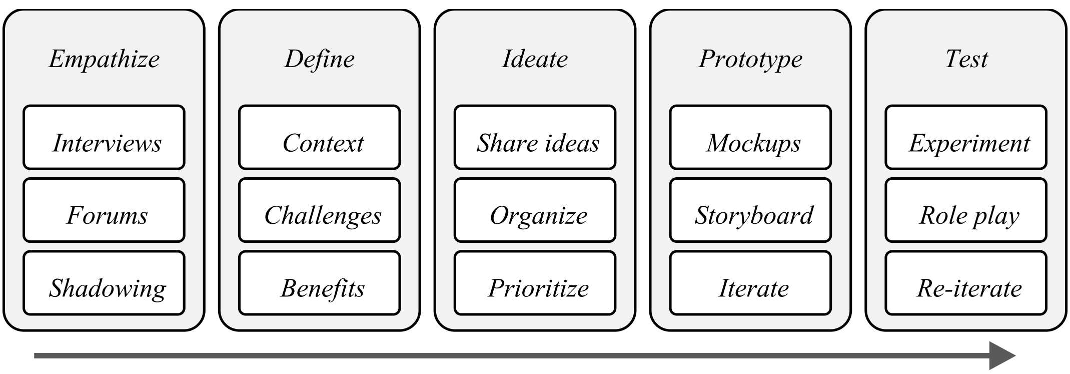


Figure 12-7 Human-centered Design Process

Human-centered design takes a systems approach by considering the surrounding context when proposing solutions rather than designing in isolation. This process can be useful for creating solutions that effectively address complex issues that interact with multiple stakeholders, like political regulations or economic policies. There is a higher chance for acceptance and usage when the concerns of the involved stakeholders are considered early on and addressed in the design with continual feedback.

Organization Models

An organization is a system of people and processes to enact change and accomplish social, economic, and other goals. An organization has a throughput, which represents inputs to outputs and changes on the environment. Organizations often contain sub-groups to carry out specific functions. For example, companies typically have a leader, like the Chief Executive Officer, who manages heads of various departments, like Operations, Finance, or Marketing, who then manages other members. These positions can be represented in an organizational chart, such as the example in Figure 12-8. Organizations often coordinate a multitude of engineering and informational technologies to create desired impacts. Organizations typically adapt over time to meet new conditions and change their underlying relations and roles, written $\Delta S_{Org} \neq 0$.

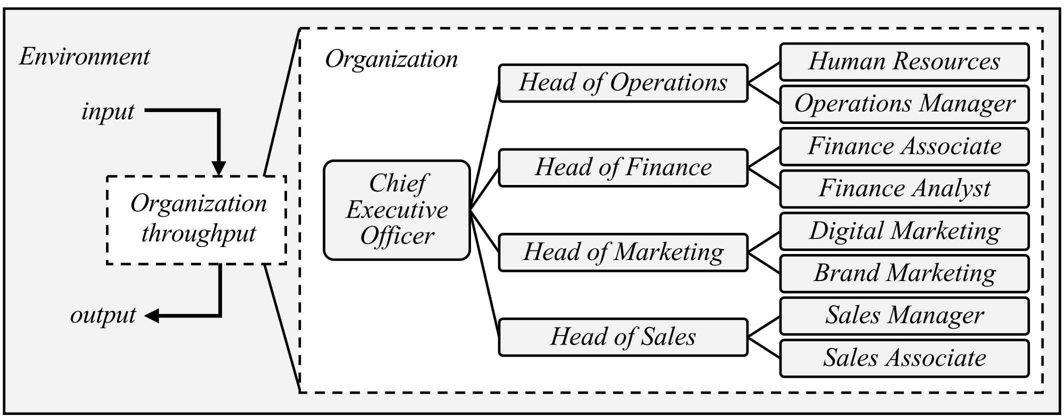


Figure 12-8 Organization Charts

Systems theory works to understand organizational models and find ways to improve outcomes. In hierarchical, or pyramid, organizations, leaders manage everyone reporting to them. A pyramid structure can be efficient, but it lacks innovation by all members and resilience when managers are not present. Another type of organizational structure is a committee where groups decide by voting, like the 193 countries in the United Nations.[249] One benefit of committees is the ability to crowd-source ideas and increase resilience, but a drawback is a lack of centralized efficiency. Another organization type is a matrix structure, where each person can have multiple managers for different topics, depending on the task at hand. A variety of models can be used, as well as hybrids of these models, to create organizations that can effectively address goals.

An enterprise architectural model categorizes an organization based on different views. For example, the framework utilized by the U.S. Department of Treasury defines four views: functional, information, organization, and infrastructural.[250] The functional view refers to the overall throughput processes associated with an organization. The information view pertains to the necessary information to carry out functions. The organization view comprises the roles and relations to carry out functions. Finally, the infrastructure is the on-the-ground tools for implementation. These various views are considered from different perspectives, like planners, owners, or builders, that create work products and enable the organization to run. Some examples of work products by view and perspective is given in Figure 12-9.

Views

Perspectives	*Functional*	*Information*	*Organization*	*Infrastructural*
Planner	*Mission & Vision Statements*	*Information Dictionary*	*Organization Chart*	*Technical Reference Model*
Owner	*Activity & Trust Model*	*Information Exchange Matrix*	*Conceptual Network*	*System Interface Level 1*
Designer	*Business Process Matrix*	*Logical Exchange Data Model*	*Logical Network Descriptions*	*System Interface Level 2*
Builder	*System Process Descriptions*	*Physical Data Model*	*Physical Network Description*	*Performance Parameters*

Work Products

Figure 12-9 Enterprise Architecture Framework

Understanding organizations through the lens of complex adaptive systems, which are systems that can learn and adapt over time, can inform effective management strategies. While a classical view of management sees organizations as machines that have a single solution to maximize efficiency, a adaptive view sees organizations as complex, evolving entities, like organisms. This means that there is not necessarily one optimal fixed way to accomplish goals, and the goals themselves may change. Instead of a machine with static rules, a systems view considers organizations as complex adaptive entities that can change rules for turning inputs to outputs, as depicted in Figure 12-10. Using complexity and adaptive based management strategies can create resilient and innovative teams that outperform strictly fixed hierarchies.[251]

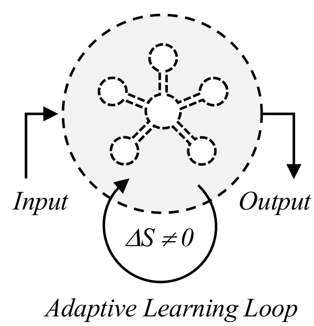


Figure 12-10 Complex Adaptive System

Systems Analysis

Systems analysis models relationships in complex systems, like business and politics, to identify optimal solutions and leverage points. Performing a systems analysis provides the opportunity to take a step back and consider the involved stakeholders and other seemingly indirect factors that contribute to a problem's nuance and complexity. After the relevant elements and relations are assessed, new proposals of how to act can be thoughtfully made.

Completing a systems analysis before implementing changes can help avoid unintended consequences of solving one problem while accidentally creating a new problem. For example, the cane toad was introduced in Australia in 1935 to control the destructive beetles that ate sugarcane crops, but the cane toad's internal poison caused an unforeseen decline in larger predators.[252] Complex systems, like ecosystems and businesses, have tightly coupled aspects where a small modification can have large unforeseen consequences. Systems analysis provides insight for understanding so-called "wicked problems", which are problems where solutions generate new problems, which can create a scenario that may be impossible to solve. It is essential to consider relevant factors to design systemic solutions.

There are many methods to support developing solutions within complex systems. The strength, weakness, opportunities, and threats, or SWOT, matrix analyzes how a decision will be helpful or harmful for both internal and external factors. A SWOT diagram is useful when making strategic business, political, and organizational decisions. Another tool for making decisions is a criteria matrix, which compares how multiple decisions will impact various criteria. The positive as well as negative results generated by a decision across relevant criteria can be considered as a whole when making decisions. These tools can inform decision making in complex systems that span multiple variables and have far reaching consequences.

	Helpful	*Harmful*
Internal	*Strengths (a,b,c,…)*	*Weaknesses (d,e,f,…)*
External	*Opportunities (g,h,i,…)*	*Threats (j,k,l,…)*

SWOT Analysis

	Decision 1	*Decision 2*	*Decision 3*
Criteria A	*1A*	*2A*	*3A*
Criteria B	*1B*	*2B*	*3B*
Criteria C	*1C*	*2C*	*3C*
Criteria D	*1D*	*2D*	*3D*

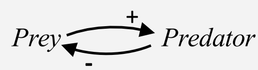
Criteria Matrix

Figure 12-11 Analysis and Decision Frameworks

Causal loop diagrams are another tool to analyze systems and devise solutions. A causal loop diagram identifies how the factors in a system influence one other through either direct relations (+ arrow) or inverse relations (- arrow). An causal loop diagram example relating to agriculture is pictured in Figure 12-12. Measures like forage biomass and environmental quality have a direct relationship (+ arrow) because when the biomass increases, the environmental quality increases. On the other hand, an increasing herd size will decrease the amount of forage biomass, which is an inverse relationship (- arrow). These diagrams identify the causal influences between many factors as well as how their relationships reinforce or balance each other.

Example 12.4
Balancing Feedback

Balancing feedback self-corrects. For example, larger prey populations support more predators (+ arrow), but increased predation reduces prey populations (- arrow).



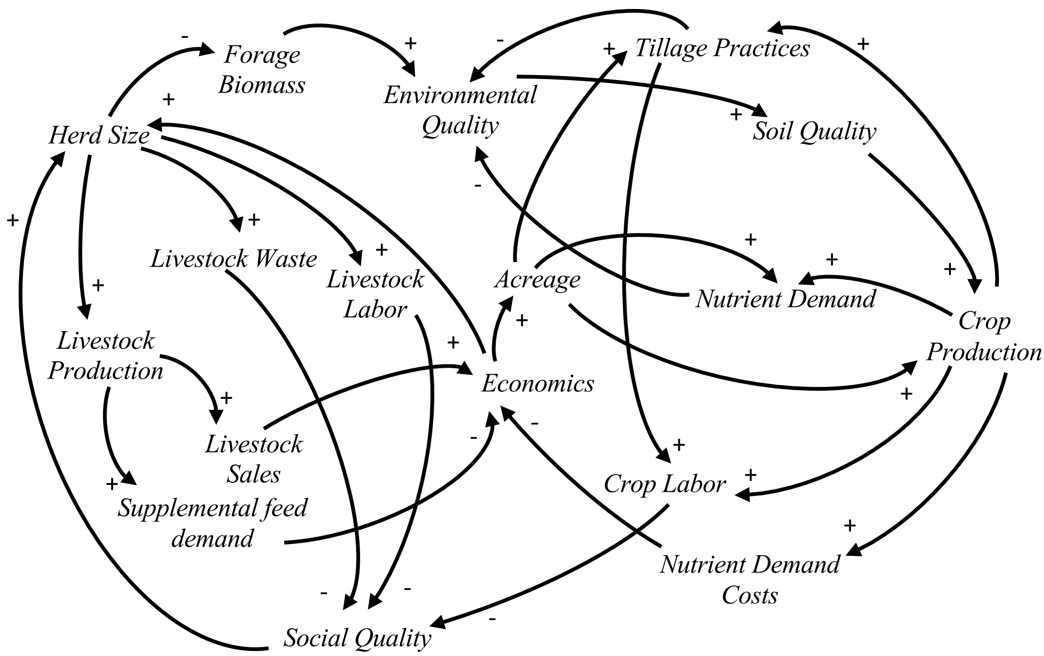


Figure 12-12 Causal Loop Diagram of Conventional Agriculture Sector

Causal loop diagrams support holistic decision making within complex systems by making it easier for stakeholders to see the big picture, develop a shared understanding of the system, and identify levers of change. Mathematical equations can also be added to model the resulting outcomes to help inform decision making. In systems analysis, complex problems cannot be thought about in terms of isolated factors, but rather as networks of interrelated connections.

There are many different strategies to effectively intervene in complex systems in our world. In the book *Thinking in Systems*, Donella Meadows identifies twelve methods to intervene in systems to create change.[253] Specific examples of these intervention strategies are named in Figure 12-13. Strategies 12 through 9 focus on short-term practical results, like changing the constants determining various flows in a system and adding buffers to support stabilizing resources. Strategies 8 through 4 prioritize highly leveraged relations, like introducing feedback, new communication flows, and self-organization. Finally, strategies 3 through 1 focus on changing the deeper goals and underlying paradigms.

12. Numbers	*Change a particular constant to influence the system: e.g., change a given tax rate to a lower or higher percentage.*
11. Buffers	*Change the size of stabilization stocks: e.g., increase food storage capacity to maintain resources over droughts and shortages.*
10. Stock-and-Flow Structures	*Add in new structures or change the structure that manages the flow in a system: e.g., add a battery to a power grid to store energy in new ways.*
9. Delays	*Change the length of time for a flow to be realized: e.g., decrease delays in business communication to promote faster adaptive action.*
8. Balancing Feedback	*Support self-balancing feedback loops: e.g., increase taxes on larger incomes to promote greater wealth equality.*
7. Reinforcing Feedback	*Support reinforcing feedback loops: e.g., use monetary savings from energy efficiency projects to invest in more efficiency projects.*
6. Information Flows	*Change the information flow within a system: e.g., require new sustainability disclosures from corporations to add information to the market.*
5. Rules	*Change incentives, punishments, and constraints: e.g., add a new law that fines unsustainable activities, like fishing in protected zones.*
4. Self-Organization	*Change the ability to create ordered states and adapt: e.g., create a digital media platform that enables new self-organized activity.*
3. Goals	*Change the purpose or function of the system: e.g., change a corporation's goal to explicitly include benefits for society and other stakeholders.*
2. Paradigms	*Change the mindset, goals, structure, rules, and parameters in systems: e.g., move from an endless growth model to a sustainability-focused economy.*
1. Transcending paradigms	*Facilitate changes to new paradigms: e.g., support education that allows evaluating the current paradigm and developing new solutions.*

Figure 12-13 Intervening in Systems

Transforming Paradigms

A society's overarching model, or paradigm, for understanding nature, can be transformed over time. The basic process of transforming scientific models is summarized in Figure 12-14, whereby a model changes to better predict collected evidence. New models are usually led by early pioneers to better account for anomalies in the current model, until a tipping point is reached which makes the new view widely accepted. Thomas Kuhn presented frameworks to understand how scientific paradigms transform in the book *The Structure of Scientific Revolutions*.

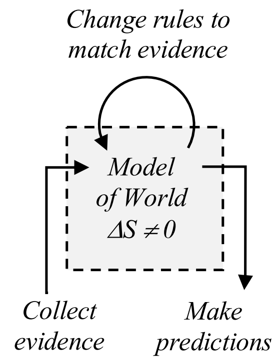


Figure 12-14 Changing Models of World

Scientific theories are changed over time through feedback between observers, formal systems, and real systems in nature. Observers can postulate a theory of nature within a formal system, which is then confirmed or denied through experiments within certain degrees of accuracy. Any formal systems can be posed, but the scientific method works to test which theories best describe nature. The scientific method iteratively compares formal systems with real-world evidence, as shown in Figure 12-15. New models of science are typically accepted because there is a larger domain of applicability and higher degrees of accuracy that makes these theories more powerful.

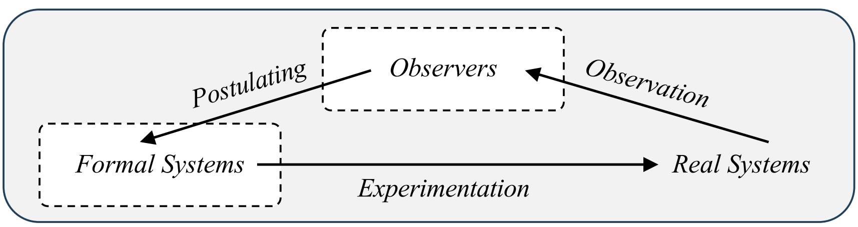


Figure 12-15 Testing Scientific Theories

While it is often believed that the current explanation of nature, or scientific paradigm, is a fixed source of truth, foundational scientific assumptions can be reformulated with new evidence. For example, it was commonly believed that the Earth, instead of the Sun, was the center of the solar system until the Copernican Revolution. Additionally, modern scientific theories, like relativity and quantum mechanics, provided completely new ways to understand fundamental assumptions of space, time, and energy. New scientific models can completely transform previous ideas. However, new scientific models should be at least as good, and ideally better, at predicting results observed in nature compared to previous models.

Periods in which scientific paradigms drastically change are called scientific revolutions. The 18th century saw the rise of mechanistic Newtonian physics in the European Scientific Revolution, along with technologies for the Industrial Revolution. The quantum mechanics revolution in the early 1900s showed that microscopic phenomena in the world are probabilistic, uncertain, and follow a wave-matter duality that requires interactions for measurements. New findings in the 1950s, like chaos theory and information theory, further challenged beliefs of predictability and reducibility and as demonstrated that different effects can emerge from the whole. In the 21st century, new sciences like climate science and regenerative design are revealing the deep interconnections between society and nature. Systems science provides a cohesive view of many of the findings of the 20th and 21st centuries and presents a paradigm of science based on complexity and interconnection. Highlighted paradigm transformations that contributed to the development of a systems-based view of science are reviewed in Figure 12-16. This graph shows approximate adoption curves of various theories that have become part of modern science. While 18th century physics supported the idea of perfect predictability and the ability to reduce any problem, modern systems science shows the world must be considered as highly interconnected, chaotic, and complex. A systems science approach is able to provide unifying view of the new paradigms that challenge the18th century parts-based view.

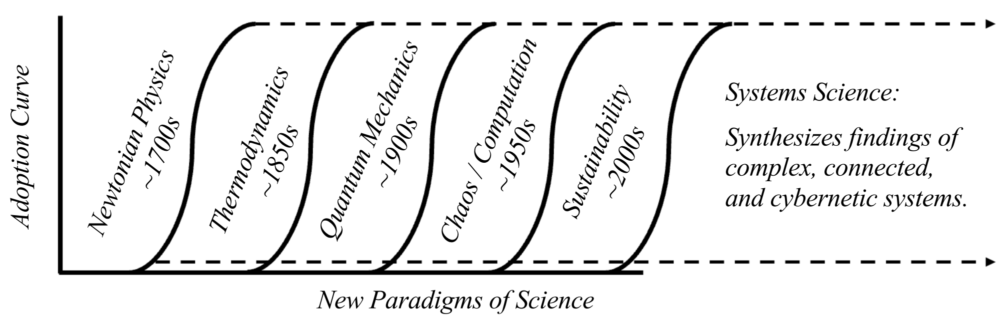


Figure 12-16 Evolution of Scientific Worldviews

The science of complex systems now supports a wide range of worldviews that are a fundamentally different from the parts-based paradigm. Fields like chaos theory show that some systems are not predictable and irreducible. Even computer simulations with simple rules cannot always be decidable to a yes or no answer. Important belief transitions supported in a systems-based paradigm—such as isolation to relations, reducibility to irreducibility, decidability to undecidability—are summarized in Figure 12-7. The new systems-based paradigm provides a more coherent understanding of science and tools to create effective solutions.

Parts-Based paradigm	→	Evidence for transition	Systems paradigm
Isolated identity	→	*Quantum wave collapse*	*Relational identity*
Predictability	→	*Three-body problem*	*Chaotic systems*
Reducibility	→	*Uncomputable numbers*	*Irreducibility*
Completeness	→	*Gödel incompleteness theorem*	*Incompleteness*
Bottom-up biology	→	*Genomic networks, epigenetics*	*Nature & nurture*
Human-nature dualism	→	*Climate change, resource limits*	*Sustainable design*

Figure 12-17 Systems Paradigm Transitions

While complex and connected systems present new evidence that challenges previous scientific paradigms, the general field of systems theory is not limited to one given model. Ancient frameworks, classical physics, and quantum mechanics are all different systems with different assumptions. Systems theory gives insight into the process of generating any given model, from simple to complex, or reducible to irreducible. The science of systems is not rigidly tied down to a specific paradigm because any set of beliefs is organized via systems. At its core, systems theory is about the process of how knowledge itself is acquired and the limitations of what can be known. Systems theory is a metatheory that transcends any one particular proposed model. With that said, the science of complex systems underscores the necessity to include irreducible emergence and interconnectivity into any comprehensive model of logic and science.

Summary

A systems-based view, which acknowledges interrelationships and complexity, provides toolsets to implement strategic change. Complex problems often arise in engineering and social systems, which can only be addressed by systemic solutions. A systems-based approach provides a new lens to consider problems within the context of the interconnected whole, which is critical to evaluating sustainability solutions that can serve a broad set of stakeholders. From a systems perspective, change is accelerated by strategically influencing the leverage points of complex networks.

Conclusion

Together, a systems-based view presents a new way to connect the disciplines of science and work towards sustainable solutions. As active and intelligent change agents, humanity can use systems thinking to refine our models of understanding nature and designing positive outcomes. While complex systems science is supported by modern evidence, the implications have yet to translate to our psychological, cultural, political, economic, and management frameworks, which often remain fixed in a past era of thought of separateness and reducibility. Integrating interconnection and complexity, which are the core attributes of the systems view, is essential to evolve our frameworks to be more insightful and effective.

Global intellect is continuously changing, and each individual will help shape the future of the world. My hope is that in the next 50 years, we can look back at our current industrial crisis as a turning point to catalyze sustainable design and thriving communities. This new turning point is based on fostering resilient connections. Each person is a part of making this a reality. Going forward, the ultimate application of systems science is through your actions to shape our complex world.

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- Figure 1-1: Examples of Systems. Word. David Shugar. December, 2018.
- Figure 1-2: Parts-Based vs. Systems-Based View. Photoshop. David Shugar. Oct, 2015.
- Figure 1-3: Chaos of Three Bodies In Gravity: Open Processing. David Shugar. November, 2022.
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The Science of Systems provides a unified approach to study all types of natural patterns and implores readers to embrace a worldview centered on connection and complexity. Complex systems challenge the view that nature can be understood as separate and predictable parts, which calls for new ways to model and interact with our interwoven world.

This interdisciplinary work studies underlying principles in logical systems and provides insights to phenomena observed in physical, informational, and biological systems. Patterns that are given particular attention include equilibrium, flux, symmetry, fractals, chaos, information, self-organization, and emergence. The book is adorned with hundreds of figures to vividly illustrate these patterns observed in nature.

The book culminates in practical applications of how systems science can be used as a tool to address many contemporary challenges, spanning environmental to socioeconomic issues. As readers navigate the complex terrain of our 21st-century challenges, ***The Science of Systems*** empowers them with a systems thinking mindset, providing insights and methods to solve problems in our interconnected and complex world.

David has an M.B.A. from Presidio Graduate School and a B.S. in Applied Physics from University of California, Santa Cruz. David's career has been focused improving the environmental and social responsibility of financial systems. David has a passion for education and has created college-level courses on interdisciplinary sciences and sustainability. This book is the culmination of a 13-year journey to synthesize concepts of systems theory, natural sciences, and sustainability.